Prediction of scour depth around bridge piers using Gaussian process

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ABSTRACT

A reliable prognostics framework is essential to prevent catastrophic failure of bridges due to scour. In the U.S., scour accounts for almost 60% of bridge failures. Currently available techniques in the literature for predicting scour are mostly based on empirical equations and deterministic regression models, like Neural Networks and Support Vector Machines, and do not predict the evolution of scour over time. In this paper, we will discuss a Gaussian process model, which includes Bayesian uncertainty for prediction of time-dependent scour evolution. We will validate the model on the experimental data conducted in four different flumes in different conditions. The robustness of the algorithm will also be demonstrated under different scenarios, like lack of training data and equilibrium scour conditions. The results indicate that the algorithm is able to predict the scour evolution with an error of less than 20% for most of the time, and 5% or less given enough training data.

Keywords: Temporal scour, prediction of scour depth, Gaussian process

1. INTRODUCTION

Many studies[1-6] have been carried out in order to understand the mechanism of scour around bridge piers. As there are many parameters that influence the scour evolution, it is very difficult to formulate a mathematical model. Due to the complex nature of the scouring process a general theory for predicting the scour was not achieved.

Recently, soft computing techniques like neural networks are being used for the prediction of scour[7-10]. Azamathulla et al.[8] presented the use of alternative neural networks to predict the scour below spillways. Bateni et al.[9] presented the use of Bayesian neural networks to predict the time-dependent scour in which both the time-dependent and equilibrium scour depth were calculated. These studies suggest that neural networks give better results compared to empirical equations[11-12]. A neural network (NN) model needs to set up different learning parameters, number of hidden layers and the number of nodes in a particular hidden layer. Also, large training sets are required to find the optimal values for the above parameters, and the NN model suffers from the problem of local minima. Pal et al.[13] developed a support vector regression based model for predicting the scour using field data. Though this model gives good predictions compared to the other methods, it does not predict the time-dependent scour.

There are only a few time-dependent scour models in the literature[5,9,14]. Hong et al.[14] developed a support vector regression based approach to predict the time-dependent scour under different sediment conditions. He was able to capture the physics of the scouring process through considering the parameters like actual and critical Froude number. Though this approach works well for the laboratory conditions, it is difficult to determine the model parameters during live bed scour where turbidity plays an important role.

All the methods described above are deterministic regression methods, and do not provide the confidence with which the predictions are made under certain conditions. The goal of this paper is to introduce the use of Gaussian process (GP), which is a probabilistic data driven approach with Bayesian uncertainty for predicting the time-dependent scour. GP has been successfully used for the structural health monitoring of aerospace components[15,16]. This paper shows the applicability of the GP algorithm for the prediction of the temporal scour

2. PARAMETER SELECTION AND DATA SETS

The scour depth (d) depends on various parameters such as velocity of the flow (V), flow depth (h), skew (S_k), pier diameter (D), median particle size (d_{50}) and gradation (σ)[12]. The underlying relationship can be written in the form:

$$d(t) = f(h, V, S_k, D, d_{50}, \sigma, t)$$
(1)

where t is the time. Parameter t facilitates the prediction of scour evolution. The type of evolution of the scour changes with different characteristics of the input parameters. The parameters such as D, P_s, S_k, d_{50}, and σ will remain same for any particular location in the streambed (near the bridge pier). Hence, in order to study the temporal evolution of the scour for a particular bridge as a function of all the above parameters, the model can be simplified by removing the parameters that remain almost constant over the given period of time. This gives a simple yet robust model for predicting the time-dependent scour depth, which will be discussed in the subsequent section. The simplified relationship can be written for the scour depth as a function of time in the form:

$$d(t) = f(h, V, t),$$
(2)

In this study, the dataset[17] containing 84 data points from experiments conducted in four different flumes was used. The characteristics of the dataset are shown in Table 1. The minimum value of the parameter is x_{min}, maximum x_{max}, mean x_{mean}, standard deviation x_{std}, variation coefficient C_{vx}, and skewness coefficient S_x.

Table 1. Characteristics of the data set[17]

Variables	x_{min}	x_{max}	x_{mean}	x_{std}	C_{vx}	S_x
***D* (mm)**	16	200	85.0075	48.2872	0.568	0.7229
***d_{50}* (mm)**	0.8	7.8	1.9261	1.7819	0.9252	1.9797
***h* (mm)**	20	600	269.7262	210.4478	0.7802	0.7385
***V* (m/s)**	0.165	1.208	0.4251	0.2698	0.6346	1.3352
***t* (min)**	200	15000	3909.3	3096.9	0.7922	1.9316
***d* (mm)**	4	318	122.75	88.744	0.723	0.5961

3. GAUSSIAN PROCESS PROGNOSIS MODEL

A Gaussian Process (GP)[18] model, which includes Bayesian uncertainty, is used for the prediction of the time-dependent scour depth. The scour depth is assumed to be a random variable, which follows the Gaussian distribution. The GP is a combination of such Gaussian distributions over the prediction time into the future. GP makes predictions by projecting the input space to the output space, by inferring their underlying non-linear relationship. Once the algorithm is trained with the input and output parameters, it can make predictions of the output parameter for unknown or new sets of input parameters. The input space and output space for the GP in the current prediction problem are shown below in the form of matrices.

$$\text{Input space} = \begin{bmatrix} h_1 & V_1 & S_{k_1} & D_1 & d_{50_1} & \sigma_1 & t_1 \\ h_2 & V_2 & S_{k_2} & D_2 & d_{50_2} & \sigma_2 & t_2 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ h_t & V_t & S_{k_t} & D_t & d_{50_t} & \sigma_t & t_t \end{bmatrix}$$

$$\text{Output space} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ . \\ . \\ d_t \end{bmatrix}$$

The posterior distribution over the predicted scour depth at time "t" (d_t) can be written as[16]:

$$f(d_t \mid D, K_t(x_i, x_j), \theta) = \frac{1}{Z}\exp\left(-\frac{d_t - \mu_{d_t}}{2\sigma^2_{d_t}}\right); \; i, j=1,\ldots,t-1$$

(3)

where Z is a normalizing constant, D = $\{x_i, d_i\}_{i=1}^{t}$ is the training set, K is the kernel matrix, θ is the set of hyper-parameters, μ_{d_t} is the mean, and $\sigma^2_{d_t}$ is the variance of the distribution, which is related to the error in the prediction. The error is attributed to the training of GP under varying conditions. The kernel function transforms the non-linear parameters to a high dimensional space where they are linearly separable. Different kernel functions[18] such as squared exponential, rational quadratic kernels were examined to determine the most suitable kernel for scour. A squared exponential kernel, which is a measure of the squared distance between the parameters, was found to work well.

The squared exponential kernel is expressed as[18]:

$$K(x_i, x_j) = \theta_1^2 \exp\left(-\frac{(x_i - x_j)^2}{\theta_2^2}\right),$$

(4)

where θ_1 and θ_2 are the hyper-parameters which govern the accuracy of the predicted values. The hyper-parameters are first initialized to a reasonable value and their optimum value is found by minimizing the negative log marginal likelihood (L) given by[18]:

$$L = -\frac{1}{2}\log|K_t| - \frac{1}{2}d_t^T K_t d_t - \frac{t}{2}\log 2\pi$$

(5)

The kernel function is first evaluated using the initialized hyper-parameters. The optimal values for the hyper-parameters are found by using the conjugate gradient descent optimization algorithm by considering "*L*" as the objective function to be minimized. The training set is updated progressively within time, as new data is available, to (i) improve the accuracy of prediction; and (ii) ensure that the model will be able to capture global and local variations in the parameters.

4. RESULTS AND DISCUSSION

The data for training was chosen such that the evolution of the scour as a function of time could be predicted. All the data was normalized before the analysis to ensure that all the parameters are equally weighted. As the total data is normalized, the hyper-parameters in equation (4) are initialized to θ_1 = 0.1, θ_2 = 0.1. Figure 1 shows the plot of the optimization of "*L*"; as the number of iterations increase the hyper-parameters reach their optimal values, thus reducing the value of "*L*". Figure 1 shows the value of "*L*" converging to the optimum value after 30 iterations.

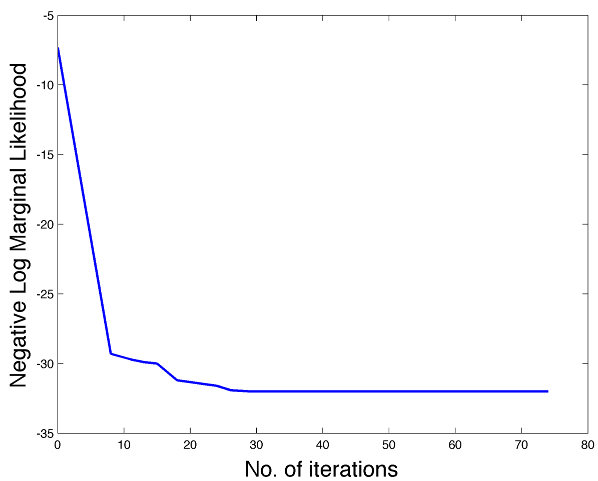

Figure 1. Optimization of the negative log marginal likelihood (*L*)

Figure 2 shows the scour depth as a function of time. Three different cases were chosen to show the adaptability and robustness of the developed algorithm. In the first case, the data about an abrupt change in the scour depth was included. The second case demonstrates the increasing accuracy of the algorithm with increasing training data. The third case demonstrates the predictive capability of the algorithm under equilibrium scour conditions. Figure 3 shows the error in prediction of the time-dependent scour depth. The grey region in Figure 2 is the 95% confidence interval (2 standard deviations) for the prediction. The algorithm is able to predict the scour depth accurately within an error of less than 5% for almost the entire scour evolution regime. The error of 8% at the final point of prediction is the result of limited training data set. However, as more data becomes available about the change, the algorithm will predict the scour depth accurately. Figure 4 (case 2) shows this phenomenon, where the model is updated dynamically to get the accurate results. The first prediction in Figure 4 is based on three training data points leading to a lower confidence level; however, as the training set is updated after every iteration, the error (Figure 5) reduces significantly. The above results show the capability of the algorithm to adapt as it gets more training data.

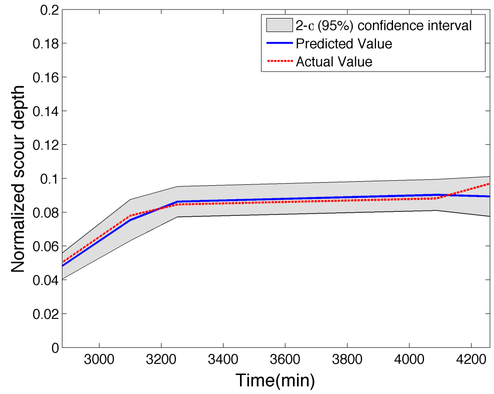

Figure 2. Scour depth as a function of time (Case1)

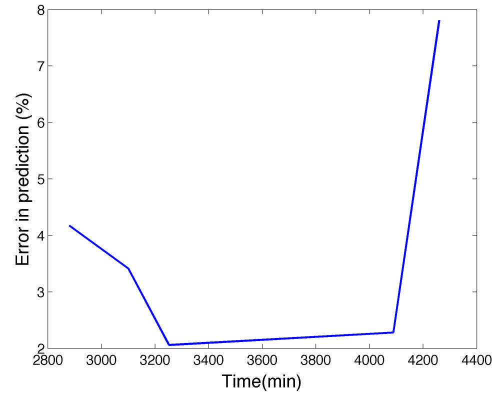

Figure 3. Error in prediction of scour depth (Case 1)

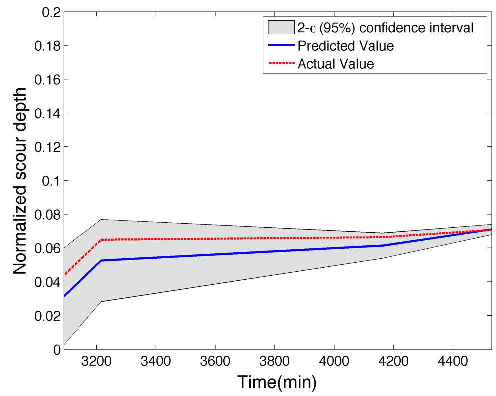

Figure 4. Scour depth as a function of time (Case 2)

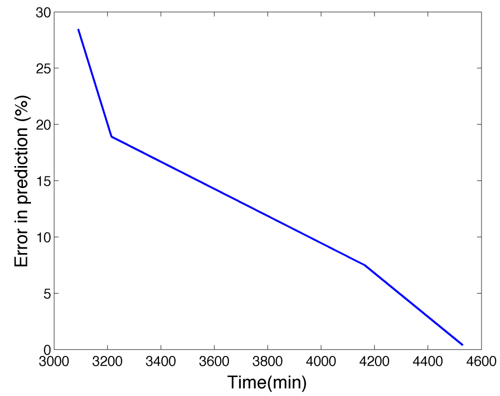

Figure 5. Error in prediction of scour depth (Case 2)

Figure 6 (case 3) shows the plot of the predicted scour-depth with time in a different flume[17]. This data was chosen to show the equilibrium scour depth. In this case, the equilibrium scour depth is achieved after 5500 minutes. Once the equilibrium scour depth is achieved, the scour remains almost constant and does not change with the input parameters. Figure 6 shows the capability of the algorithm to capture this phenomenon. During the equilibrium phase, the error in prediction is less than 2%.

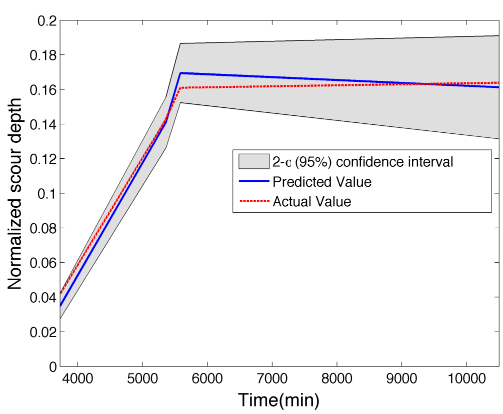

Figure 6. Scour depth as a function of time (Case 3)

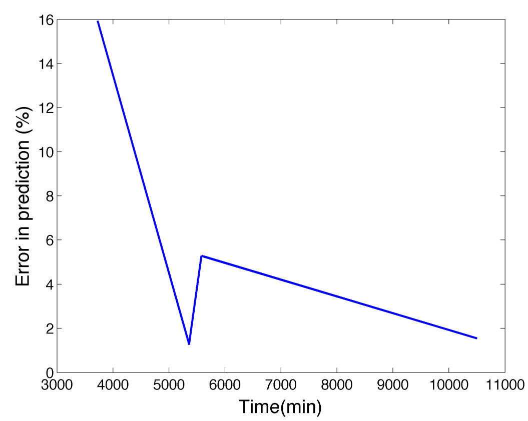

Figure 7. Error in prediction of scour depth (Case 3)

The above three cases were carefully chosen and presented to show the adaptability and robustness of the GP algorithm under different conditions. Figure 8 shows the plot of the actual scour versus predicted scour for all the dataset. Out of the 84 points in the dataset, 27 points, which were chosen from different flumes under different flow conditions, were used for prediction. All the predicted points in Figure 8, except two, lie within the ±25% error lines. Out the points lying inside the ±25% error lines, most of them are very close to the centerline, which implies accurate predictions.

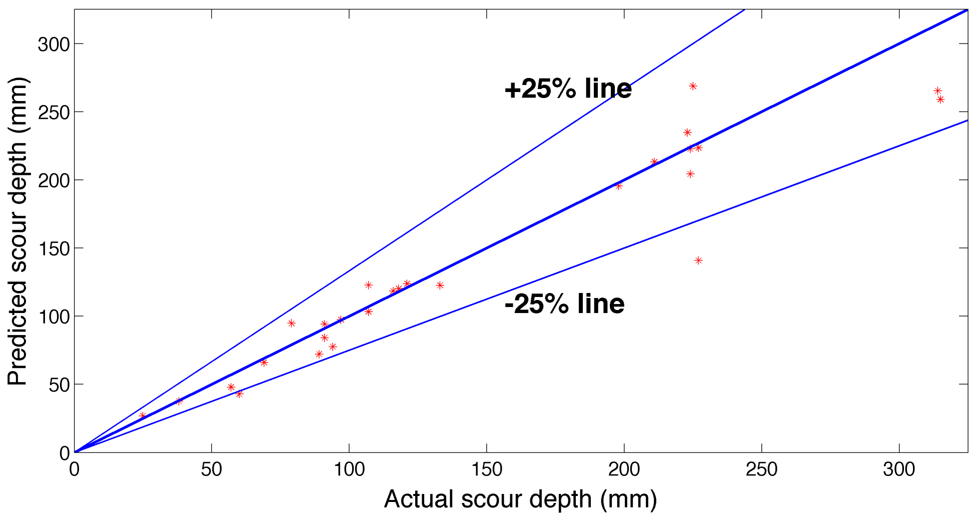

Figure 8. Actual scour vs. predicted scour using the Gaussian Process algorithm

Table 2 shows that 92.5% of the points are predicted with an error of less than or equal to 20%. Out of these points, which had the error in between 10% and 20%, are the points that had the least amount of training data (≤ 4 training data points). A coefficient of determination of 0.9821 was achieved for the training set and a value of 0.9016 was achieved for the test data.

Table 2. Number of points predicted with a certain range of error.

Error (E) (%)	**Number of points predicted**
E < 5	12
5 < E < 10	5
10 < E < 20	8
E > 20	2

5. CONCLUSIONS

A probabilistic Gaussian process based algorithm for predicting the time-dependent scour has been developed. Three different scenarios were demonstrated to test the capability of the algorithm. In the first case, the data containing sudden increase in scour was considered. The algorithm was able to predict this phenomenon with an error of 8%. In the second case, the adaptability of the algorithm with increasing training data is shown. The error in the prediction decreases asymptotically as more training data becomes available. In the third case the data was chosen such that the scour has reached the equilibrium value where it doesn't change with the varying input conditions. The GP algorithm was able to capture this phenomenon and predicted a constant scour depth during this period. Out of the 84 data points available, 27 data points were used for testing the algorithm. A coefficient of determination of 0.9821 was achieved for the training data and a value of 0.9016 was achieved for the testing set.

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REFERENCES

[1] Ettema, R., Melville, B. and Barkdoll, B., "Scale effect in pier-scour experiments", Journal of Hydraulics Engineering 124(6), 639-42 (2001).
[2] Lim, SY. and Cheng, NS., "Prediction of live-bed scour at bridge abutments", Journal of Hydraulics Engineering 124, 635-42 (1998).
[3] Melville, B. and Sutherland, AJ., "Design method for local scour at bridge piers", Journal of Hydraulics Engineering 114(10), 1210-26 (1988).
[4] Melville, B., "Pier and abutment scour: integrated approach", Journal of Hydraulics Engineering 123, 125-36 (1997).
[5] Mia, F. and Nago, H., "Design method of time dependent local scour at circular bridge pier", Journal of Hydraulics Engineering 129(6), 420-7 (2003).
[6] Parola, AC., Mahavadi, SK., Brown, BM. And Elkhoury A., "Effect of rectangular foundation geometry on local pier scour", Journal of Hydraulics Engineering 122, 35-40 (1996).
[7] Firat, M. and Gungor, M., "Generalized regression neural networks and feed forward neural networks for prediction of scour depth around bridge piers", Advances in Engineering Software 40(8), 731-737 (2009).
[8] Azmathulla, H.Md., Deo, M.C. and Deolalikar, P.B., "Alternative neural networks to estimate the scour below spillways", Advances in Engineering Software 39(8), 689-798 (2008).
[9] Bateni, S., Jeng, D. and Melville, B., "Bayesian neural networks for prediction of equilibrium and time-dependent scour depth around bridge piers", Advances in Engineering Software 38(2), 102-111 (2007).
[10] Bateni, S., Borghei, S. and Jeng, D., "Neural network and neuro fuzzy assessments for scour depth around bridge piers", Engineering Applications of Artificial Intelligence 20(3), 401-414 (2007).
[11] McIntosh, J.L., "Use of scour prediction formulae", Proceedings of the Bridge Scour Symposium, McLean, VA, Federal Highway Administration Research Report FHWA-RD-90-035, 78-100 (1989).
[12] Mueller, D. and Wagner, C., "Field Observations and Evaluations of Streambed Scour at Bridges", Federal Highway Administration, US Department of Transportation, publication no. FHWA-RD-03-052 (2005).
[13] Pal, M., Singh, N.K. and Tiwari, N.K., "Support vector regression based modeling of pier scour using field data", Engineering Applications of Artificial intelligence 24(5), 911-916 (2011).
[14] Hong, J., Goyal, M., Chiew, Y. and Chua, L., "Predicting time-dependent pier scour depth with support vector regression", Journal of Hydrology, 468-469, 241-248 (2012).
[15] Mohanty, S., Das, S., Chattopadhyay, A. and Peralta, P., "Gaussian process time series model for life prognosis of metallic structure", Journal of Intelligent Material Systems and Structures 20(8), 887-896 (2009).
[16] Mohanty, S., Chattopadhyay, A. and Peralta, P., "Adaptive residual useful life estimation of a structural hotspot", Journal of Intelligent Material Systems and Structures 21(3), 321-335 (2010).

[17] Melville, B. and Chiew, Y., "Time scale for local scour at bridge piers", Journal of Hydraulic Engineering 125(1), 59-65 (1999).
[18] Rasmussen, C.E. and Williams, C.K.I., "Gaussian processes for machine learning", The MIT Press, Cambridge, MA (2006).