Variability Characterization and Stochastic Multiscale Modeling of Composite Materials

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Research supported by Army Research Office

AIMS ADAPTIVE INTELLIGENT MATERIALS & SYSTEMS CENTER

Objectives:

- **Develop multiscale modeling framework for microstructural uncertainty quantification**
- **Quantify effect of microscale variability on damage and global behavior**

Project Objectives

Stochastic Multiscale Modeling

- Uncertainty in micromechanics modeling
- Damage progression
- Integrated structural scale modeling
- Wave propagation modeling in composites

Experiments and Validation

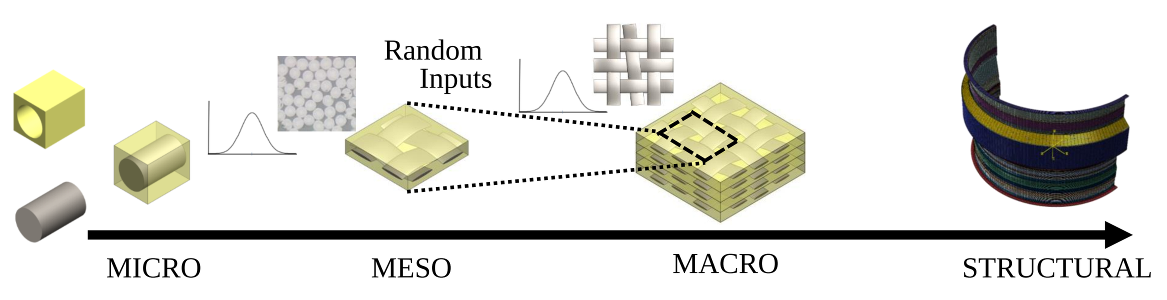
- Microstructural variability
- Constituent material property variability
- Investigate various failure modes due to impact and fatigue testing

Outcome

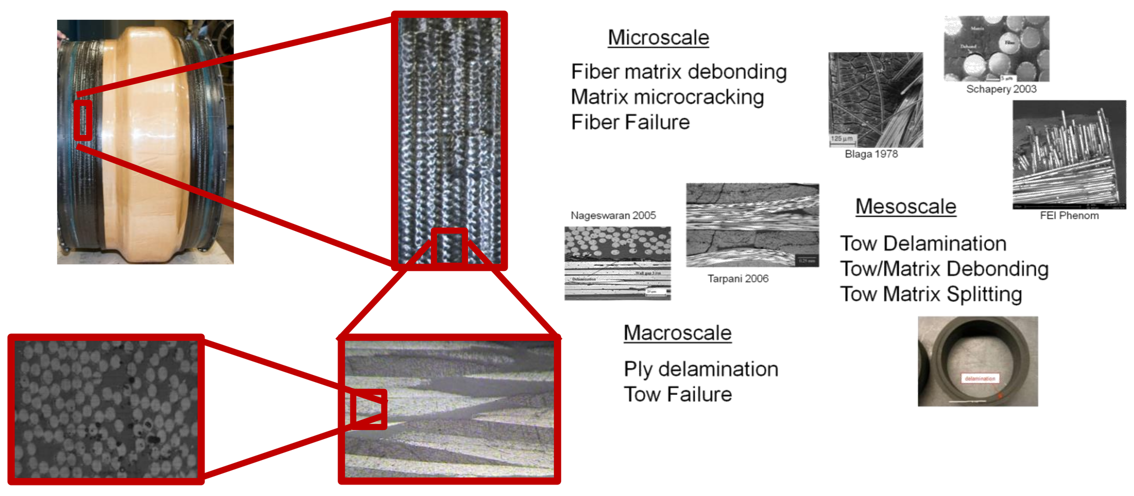
- Material characterization and effect on microstructure uncertainty
- Stochastic modeling of damage progression
- Modeling uncertainties within a multiscale framework

Multiscale Modeling

- Stochastic physics-based multiscale model key to understanding damage initiation & progression in heterogeneous systems
- Model output to reduce resources required for experiments
- Propagation & effect of uncertainty across length scales essential to understanding their effects at the macroscale
- Integration of multiscale models with FEA



Effects of Microstructure on Uncertainty

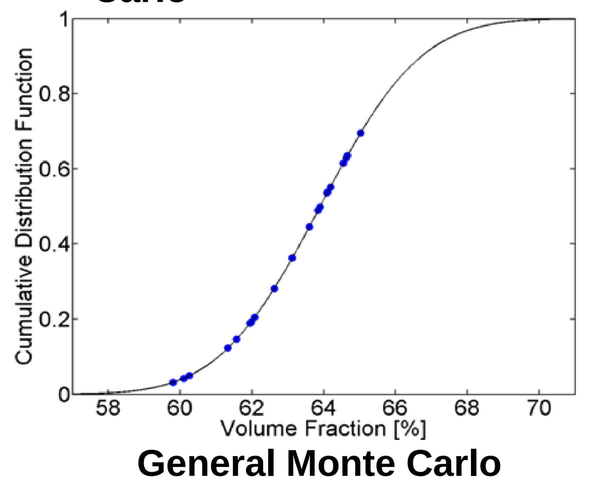


- Microstructural & length scale effects
- Damage initiation, progression, & evolution
- Multiple damage & failure mechanisms
- Rate dependence, thermal, & environment effects

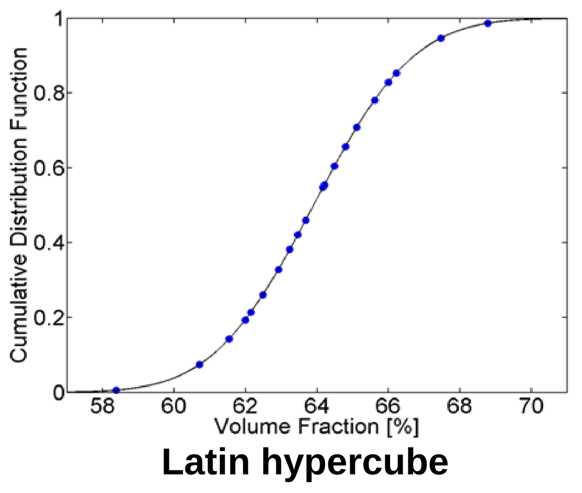
Global behavior & local phenomena such as damage influenced by uncertainty effects at lower length scales

Stochastic Methodologies

- Using Latin Hypercube sampling in Monte Carlo simulations
  - Discretize the statistical distributions into intervals
  - Randomly choose points within those discretized intervals
- Compare Latin hypercube method with general Monte Carlo

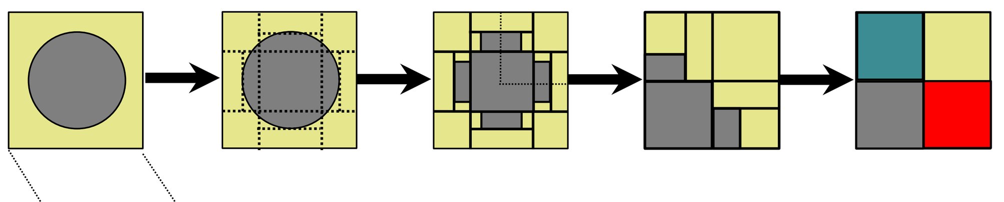


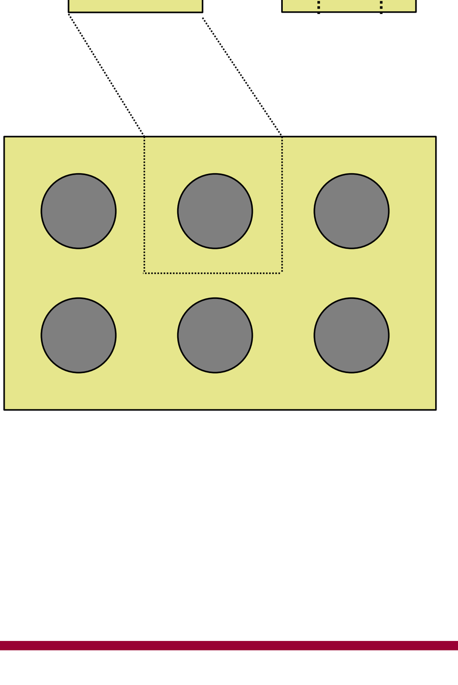
General Monte Carlo



Latin hypercube

3D Sectional Model

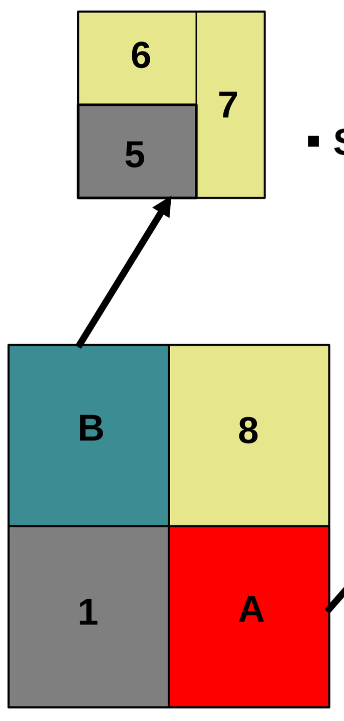


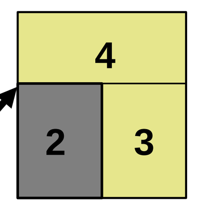


- Unit cell is discretized into sections
- Quarter symmetry assumed for the unit cell
- Sectional approach for transverse isotropy & computational efficiency

3D Sectional Model

- First Level
  - Sections B and A consist of two matrix subcells and 1 fiber subcell
  - Continuity assumptions for subcells for 3D stress and strains
- Second Level
  - Continuity assumptions between the groups (A, B, 1, 8)





Fiber: $\dot{\boldsymbol{\sigma}}_f = \mathbf{C}_f \dot{\boldsymbol{\varepsilon}}_f$

Matrix:

$$\dot{\boldsymbol{\sigma}}_m = \mathbf{C}_m \left(\dot{\boldsymbol{\varepsilon}}_m - \dot{\boldsymbol{\varepsilon}}_m^I \right)$$

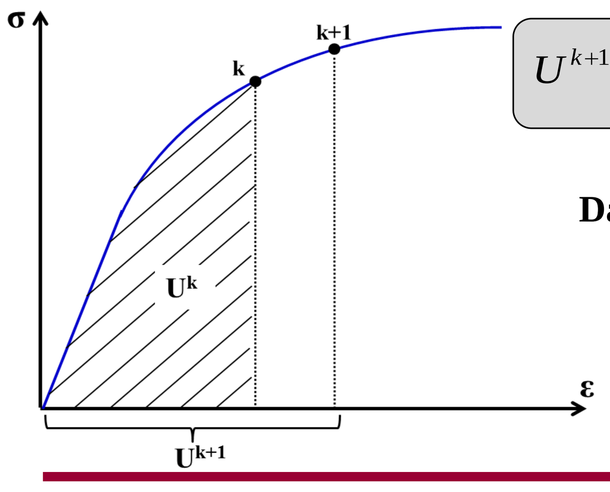
$$\dot{\boldsymbol{\varepsilon}}_m^I = 2D_0 \exp\left[-\frac{1}{2}\left(\frac{Z}{\sigma_e}\right)^{2n} \right] \left(\frac{\mathbf{S}}{2\sqrt{J_2}} + \alpha \boldsymbol{\delta} \right)$$

Incremental Damage Theory

- Theory based on work potential model (Schapery, 1990)

$$U = W_E + W_S$$

- Where U is the total potential work, W_E is the elastic strain energy, and W_S is the energy for structural change
- Incorporated incremental Schapery theory within stochastic sectional micromechanics
- Moduli are degraded using the "e" and "g" factors which depend on the microdamage variable, S



$$U^{k+1} = U^k + \frac{1}{2} d\varepsilon_{ij} \left(2\sigma_{ij}^{k+1} - d\sigma_{ij} \right)$$

Damage Functions:

$$E_{22} = E_{220} e_s(S)$$
$$G_{12} = G_{120} g_s(S)$$

Composite Failure Theories

- Microfailure Theory (Max. Stress)
  - Failure of individual subcells
  - Rate dependent failure parameters for matrix subcells
- Macroscale Failure Theory
  - Failure of unit cell
  - Modified Hashin-Rotem failure criterion to incorporate shear stress effects
  - Four individual failure modes

Tensile Fiber Mode

$$\begin{cases} \left(\dfrac{\sigma_{11}}{\sigma_A^+}\right)^2 + \dfrac{1}{\tau_A^2}\left(\sigma_{12}^2 + \sigma_{13}^2\right) = 1 \\ \sigma_{11} > 0 \end{cases}$$

Compressive Fiber Mode

$$\begin{cases} \left(\dfrac{\sigma_{11}}{\sigma_A^-}\right)^2 + \boxed{\dfrac{1}{\tau_A^2}\left(\sigma_{12}^2 + \sigma_{13}^2\right)} = 1 \\ \sigma_{11} < 0 \end{cases}$$

Tensile Matrix Mode

$$\begin{cases} \dfrac{1}{\sigma_T^{+2}}\left(\sigma_{22} + \sigma_{33}\right)^2 + \dfrac{1}{\tau_T^2}\left(\sigma_{23}^2 - \sigma_{22}\sigma_{33}\right) + \dfrac{1}{\tau_A^2}\left(\sigma_{12}^2 + \sigma_{13}^2\right) = 1 \\ \sigma_{22} + \sigma_{33} > 0 \end{cases}$$

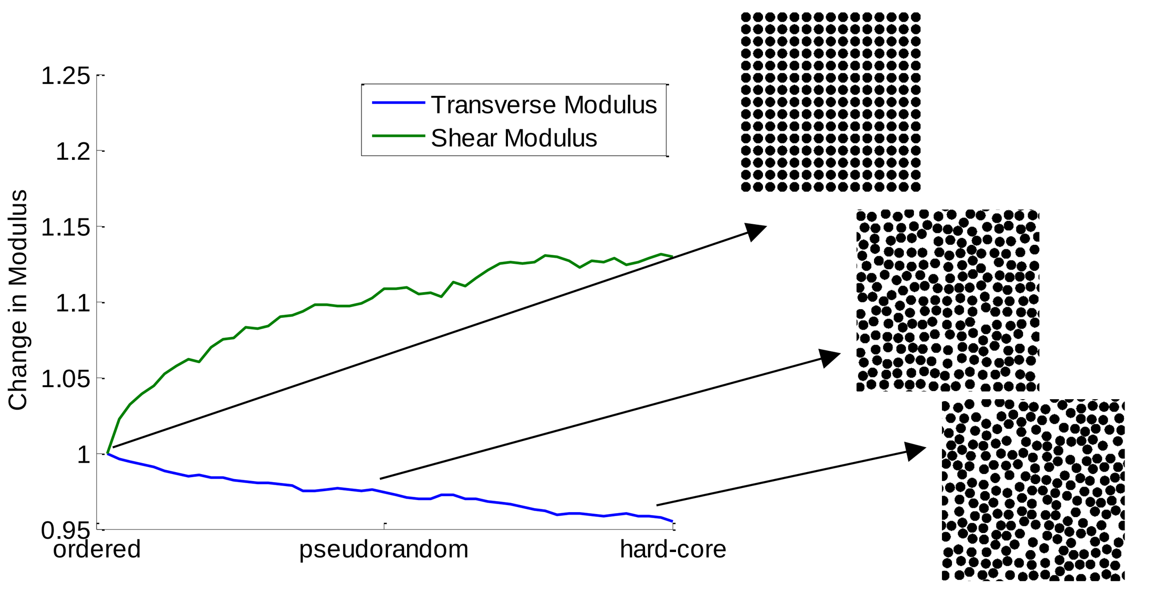
Compressive Matrix Mode

$$\begin{cases} \dfrac{1}{\sigma_T^-}\left[\left(\dfrac{\sigma_T^-}{2\tau_T}\right)^2 - 1\right]\left(\sigma_{22} + \sigma_{33}\right) + \dfrac{1}{4\tau_T^2}\left(\sigma_{22} + \sigma_{33}\right)^2 + \dfrac{1}{\tau_T^2}\left(\sigma_{23}^2 - \sigma_{22}\sigma_{33}\right) + \dfrac{1}{\tau_A^2}\left(\sigma_{12}^2 + \sigma_{13}^2\right) = 1 \\ \sigma_{22} + \sigma_{33} < 0 \end{cases}$$

Spatial Randomness

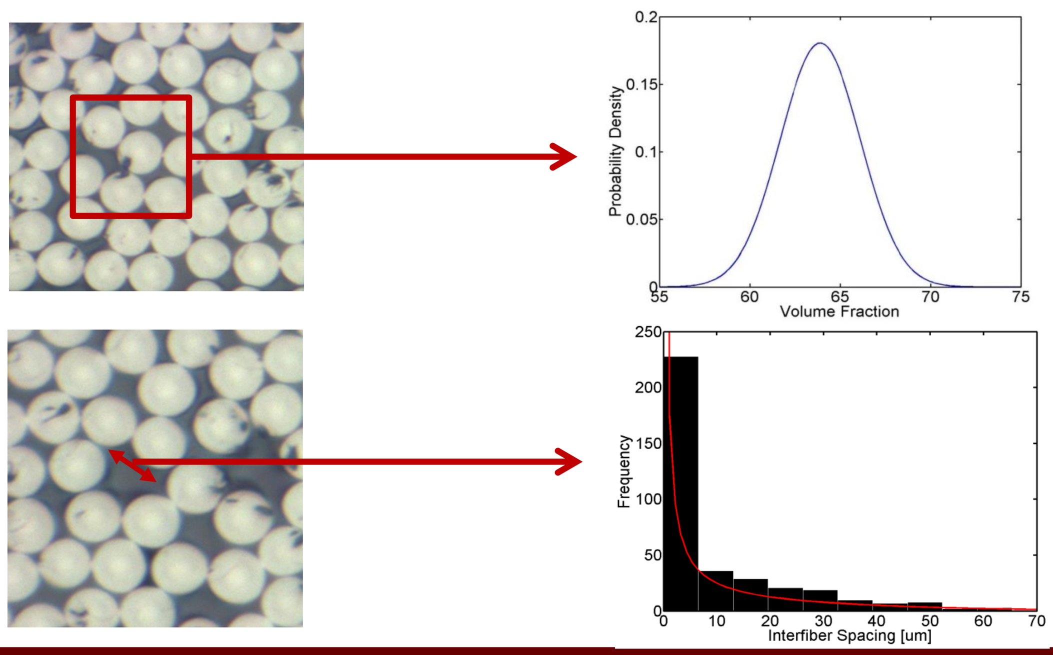
As spatial randomness increases:

- Transverse tensile modulus decreases by ~5%
- Shear modulus increases by ~15%

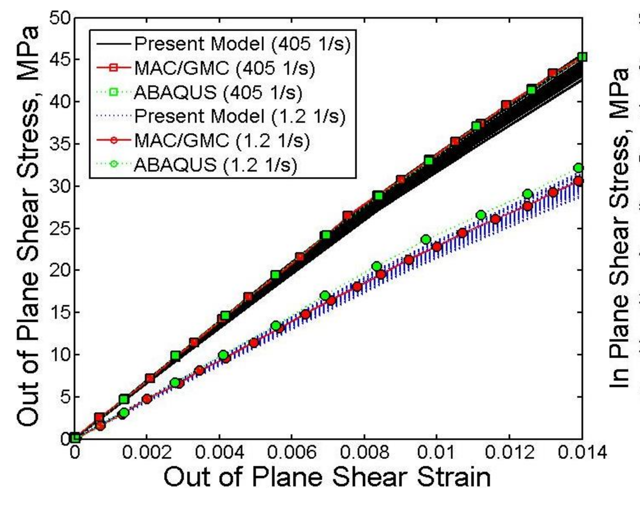


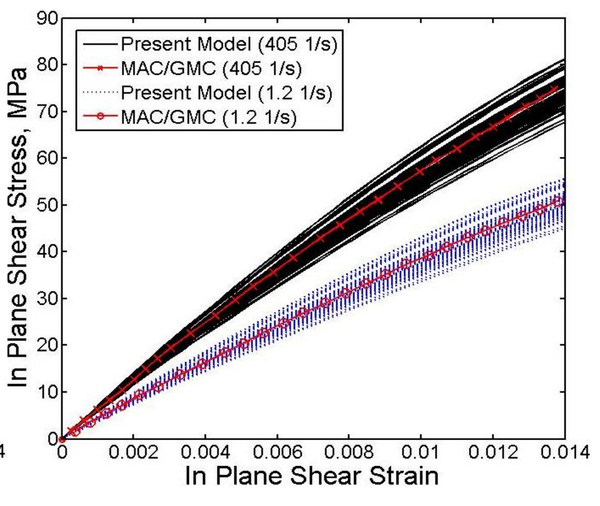
Characterization Results

- Fiber volume fraction, fiber diameter, & spacing statistical distribution functions from optical microscopy
- Results from microscopy analysis of the polymer matrix composite used as random inputs in the multiscale models



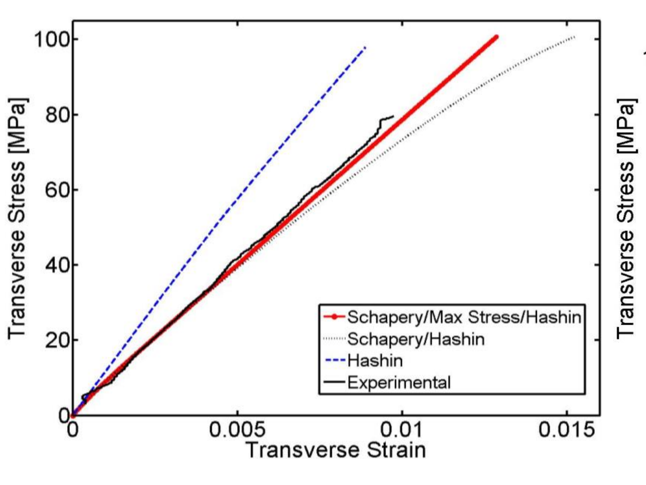
Model Validation

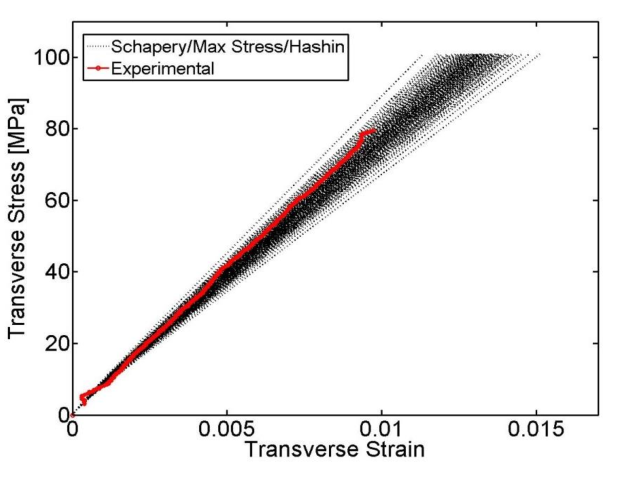




- Present model with Hashin theory captures MAC/GMC & ABAQUS responses for both strain rates
- In-plane shear response indicates a larger sensitivity to random volume fraction compared to out-of-plane shear response

Failure Theory Comparison





- Deterministic behavior comparing different combination of failure & damage theories
- Stochastic response curves show better correlation with the experimental data