Modeling and Optimization of Passively Damped Adaptive Composite Structures

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ABSTRACT: A smart structural model is used to optimally determine both the placement of piezoelectric actuators and parameters describing associated electrical components in a passively damped structure. The technique utilizes a recently developed coupled piezoelectric-mechanical theory to analytically determine the response of arbitrary structures with piezoelectric materials and attached electrical circuitry. The theory simultaneously models both the structural and the electrical components, and the complex state of strain that may exist in the piezoelectric patches, thereby providing accurate mechanical and electrical response. A robust multiobjective optimization procedure is developed to design the passive system for simultaneous damping of several critical modes of interest. The influence of stacking sequence in augmenting passive damping can also be examined by including ply orientations as design variables. Since the optimization problem now involves both continuous and discrete design variables, a hybrid optimization technique is used that allows the inclusion of both types of design variables. Also, since multiple design objectives are introduced, the Kreisselmeier–Steinhauser function approach is used allowing the multiple and conflicting design objectives and constraints to be combined into a single unconstrained function. Results demonstrate the ability of this technique to determine the optimal tuning parameters for damping of multiple modes of vibration using a limited number of piezoelectric actuators.

Key Words: piezoelectric, smart structures, passive damping, optimization

INTRODUCTION

PIEZOELECTRIC actuators are often used to control the dynamic response of structures, however accurate description of the interaction between the structural characteristics and the associated electrical circuitry is an often overlooked aspect in the design of smart structural systems. Piezoelectric materials (PZT) add additional stiffness to the structure, but this stiffness is dependent upon the electrical circuitry attached to each PZT. Electrical components can contain, store and dissipate both potential and kinetic energy, and PZT allows for transformation between mechanical and electrical energy. For efficient performance of a smart structural system, it is necessary to address all of these issues associated with the entire system to accurately capture the response to a set of arbitrary external conditions. By simultaneously modeling both the structure and the electrical components, accurate calculation of mechanical and electrical response is possible.

A common application of the electrical interaction with the structural deformation is in the design of passive electrical damping systems. Passive damping circuits have the ability to convert mechanical strain energy into electrical energy in the PZT and then dissipate this energy as heat in resistors. However, to be most effective, these circuits must be tuned to damp out particular modes. Hagood et al. (Hagood and Crawley, 1991; Hagood and Von Flotow, 1991) conducted significant investigation in the areas of passive electrical damping and self-sensing actuators using coupled equations similar to those used in this work. Wu (Wu, 1996, 1998; Wu and Bicos, 1997) has developed several techniques for passive damping circuits and has demonstrated methods for damping out multiple vibrational modes. Other research (Kahn and Wang, 1994; Tsai and Wang, 1996, 1999) has demonstrated how passive damping circuits and active control can be combined in active–passive hybrid piezoelectric networks to enhance damping capability. This research also made use of optimization techniques to choose electrical parameters to maximize control authority (Tsai and Wang, 1996). However, in all of these works, although significant effort was made in addressing the electrical aspects of the damping systems, very simple structural models, that do not take into account the complex state of strain that may exist in the piezoelectric patches, were used. In addition, the placement of the piezoelectric patch has not been considered concurrently with the design of the electrical system. Much of the work

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available in the literature addressed the vibration of cantilevered beams and plates where a piezoelectric patch located near the root can be effective in controlling all of the lower order modes. Structures with other boundary conditions, where optimal location varies for each mode and are not intuitively obvious have not been examined in detail.

The objective of this work is to demonstrate how multidisciplinary optimization (MDO) techniques can be utilized to optimize both structural and electrical aspects of an adaptive structural system. To simultaneously optimize multiple performance requirements, the Kreisselmeier–Steinhauser (K–S) function approach (Wren, 1989; Sethi and Striz, 1997) is used. The K–S technique is a multiobjective optimization procedure that combines all the objective functions and the constraints to form a single unconstrained composite function to be minimized. Then an unconstrained solver is used to locate the minimum of the composite function. The advantage of this method is that it does not rely on arbitrary weight factors to combine multiple objective functions although they can be used in cases where a designer wishes to emphasize specific design criteria (Rajadas et al., 2000).

Optimization of an integrated structural and electrical system involves both discrete and continuous variables. Gradient based methods are generally ineffective in the optimization of discrete variables and nongradient based techniques such as genetic algorithms (GA) and simulated annealing (SA) can be computationally very expensive (Belegundu and Chandrupatla, 1999). A hybrid method (Seeley et al., 1996) has been developed that uses a discrete search combined with a gradient based technique for the continuous design variables. The use of this technique provides significant improvement in computational efficiency over traditional discrete searches by using a combinatorial search algorithm that allows the use of gradients for the continuous variables, while using a discrete search technique for the discrete design variables. The optimization method used in this work is very similar to the hybrid method of Seeley, with the major difference being that the discrete search and the gradient based search are more independent due to the nature of this particular problem. Since, the present model assumes that only the electrical components to be continuous parameters the gradient based search essentially amounts to finding the optimal set of electrical components for a given structural configuration.

In this paper, a method is presented for the simultaneous design of electro-mechanical parameters of a smart structural system using an efficient and accurate modeling technique and multiobjective optimization procedure. This allows optimization of composite plates, including stacking sequence, regardless of the boundary conditions or plate geometry.

COUPLED PIEZOELECTRIC-MECHANICAL FORMULATION

A recently developed two-way coupled piezoelectric-mechanical theory (Thornburgh and Chattopadhyay, 2002) is used to model composite plates with piezoelectric actuators. The coupled equations are formulated in terms of the mechanical strain, ε_{kl}, and the electric displacement, D_k, as opposed to the strain and electric field.

$$\sigma_{ij} = c^{D}_{ijkl}\varepsilon_{kl} - h_{kij}D_k \tag{1}$$

$$E_i = -h_{ikl}\varepsilon_{kl} + \beta^{S}_{ik}D_k \tag{2}$$

where c^{D}_{ijkl}, h_{ijk}, and β^{S}_{ij} are the open circuit elastic and zero strain dielectric constants, respectively, and σ_{ij} and E_i are the stress tensor and the electric field vector. The coefficient h_{ijk} represents the coupling between the strain and the electric displacement. In matrix form these are written as

$$\sigma = \mathbf{C^D}\varepsilon - \mathbf{hD} \tag{3}$$

$$\mathbf{E} = -\mathbf{h^T}\varepsilon + \boldsymbol{\beta}^{\mathbf{S}}\mathbf{D} \tag{4}$$

Using this formulation, the electric displacement ($\mathbf{D}$) can be taken as constant through the thickness of the PZT, thus ensuring conservation of charge on each of the electrodes.

The equations of motion can be formulated using a variational approach and Hamilton's Principle similar to that by Tiersten (1967). The variational principle between times t_0 and t, for the piezoelectric body of volume V can be written as follows

$$\delta\Pi = 0 = \int_{t_o}^{t}\int_{V}\left[\delta\left(\frac{1}{2}\rho\dot{\mathbf{u}}^{\mathbf{T}}\dot{\mathbf{u}}\right) - \delta\,\mathbf{H}(\varepsilon,\mathbf{D})\right]dVdt + \int_{t_o}^{t}\delta\mathbf{W}dt \tag{5}$$

where the first term represents the kinetic energy, the second term the electric enthalpy and $\delta\mathbf{W}$ is the total virtual work done on the structure. The terms $\mathbf{u}$ and ρ correspond to the mechanical displacement and density, respectively. The electric enthalpy is given by

$$\mathbf{H}(\varepsilon,\mathbf{D}) = \frac{1}{2}\varepsilon^{\mathbf{T}}\mathbf{C^D}\varepsilon - \varepsilon^{\mathbf{T}}\mathbf{hD} + \frac{1}{2}\mathbf{D^T}\beta^{\mathbf{S}}\mathbf{D} \tag{6}$$

The work done by body forces ($\mathbf{f_B}$), surface tractions ($\mathbf{f_S}$) and electrical potential (ϕ) applied to the surface of the piezoelectric material can be expressed by

$$\delta\mathbf{W} = \int_V \delta\mathbf{u^T f_B}dV + \int_S \delta\mathbf{u^T f_S}dS + \int_S \delta\mathbf{D^T}\phi dS \tag{7}$$

Equations (5)–(7) provide the equations of motion for the piezoelectric body. Using a finite element formulation, they can be expressed in matrix form as follows

$$\begin{bmatrix} \mathbf{M_u} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_\mathbf{e} \\ \ddot{\mathbf{D}} \end{Bmatrix} + \begin{bmatrix} \mathbf{C_u} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_\mathbf{e} \\ \dot{\mathbf{D}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K_{uu}} & \mathbf{K_{uD}} \\ \mathbf{K_{Du}} & \mathbf{K_{DD}} \end{bmatrix} \begin{Bmatrix} \mathbf{u_e} \\ \mathbf{D} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F_u} \\ \mathbf{F_D} \end{Bmatrix} \tag{8}$$

where $\mathbf{u_e}$ is the nodal displacement vector and $\mathbf{D}$ is the vector of nodal electrical displacements. The matrix $\mathbf{M_u}$ is the structural mass matrix and $\mathbf{C_u}$ is the structural damping matrix. The matrix $\mathbf{K_{uu}}$ is the mechanical stiffness matrix, $\mathbf{K_{DD}}$ is the electrical stiffness matrix, and $\mathbf{K_{uD}}$ and $\mathbf{K_{Du}}$ are the stiffness matrices due to piezoelectric-mechanical coupling. The vectors $\mathbf{F_u}$ and $\mathbf{F_D}$ are the force vectors due to mechanical and electrical loading. A structural damping matrix $\mathbf{C_u}$ is added to incorporate damping. The nature of the damping matrix can be chosen to meet the needs of the user. The stiffness matrices are further defined in the literature (Thornburgh and Chattopadhyay, 2002).

The absence of any electrical inertia or damping terms in Equation (8) is a result of only considering the mechanical aspects of the system. In modeling an integrated smart structural system, it is necessary to include additional terms associated with the electrical components. For a simple LRC circuit, the linear equations of motion can be written as follows

$$\mathbf{M_q}\ddot{\mathbf{q}}_\mathbf{e} + \mathbf{C_q}\dot{\mathbf{q}}_\mathbf{e} + \mathbf{K_q}\mathbf{q_e} = \mathbf{F_q} \tag{9}$$

then these equations can be directly combined with Equation (8). The charge flows ($\mathbf{q_e}$) in the electrical circuit represents the net charge flow into or out of each piezoelectric device. The net charge flow corresponds to the integral of the electric displacement over the area of piezoelectric patch, and since a finite element formulation is used, the net charge is equal to the product of the nodal electrical displacements and an area matrix, $\mathbf{A_q}$. This method allows modeling of the interaction between the electric displacement and the electric circuit regardless of the number of elements the piezoelectric device occupies. This combination results in a completely coupled electrical-mechanical system of the following form.

$$\begin{aligned} &\begin{bmatrix} \mathbf{M_u} & \mathbf{0} \\ \mathbf{0} & \mathbf{A_q^T M_q A_q} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_\mathbf{e} \\ \ddot{\mathbf{D}}_\mathbf{e} \end{Bmatrix} + \begin{bmatrix} \mathbf{C_u} & \mathbf{0} \\ \mathbf{0} & \mathbf{A_q^T C_q A_q} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_\mathbf{e} \\ \dot{\mathbf{D}}_\mathbf{e} \end{Bmatrix} \\ &+ \begin{bmatrix} \mathbf{K_{uu}} & \mathbf{K_{uD}} \\ \mathbf{K_{Du}} & \mathbf{K_{DD}} + \mathbf{A_q^T K_q A_q} \end{bmatrix} \begin{Bmatrix} \mathbf{u_e} \\ \mathbf{D_e} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F_u} \\ \mathbf{F_D} + \mathbf{A_q^T F_q} \end{Bmatrix} \end{aligned} \tag{10}$$

It has been shown in the literature that it is possible to reduce a system using the mode shapes rather than work with the full finite element model (Thornburgh and Chattopadhyay, 2002). The system can be partially reduced using the structural mode shapes if the assumption is made that the mode shapes for the coupled system can be expressed in terms of the open circuit mode shapes. This assumption is generally quite reasonable since changes in the elastic stiffness of PZTs due to open circuiting or short circuiting cause only modest shifts in natural frequency and very small alterations in the mode shapes. The structural stiffness introduced by the PZTs is generally small compared to the stiffness of the host structure and the difference between open circuit and short circuit stiffness is only around twenty percent. This combined with the fact that PZTs typically cover a fraction of the structural surface indicates that the differences in mode shapes between the open circuited condition and the actual structure will be generally small and localized.

First, the coupled system is reduced so that the open circuit eigenvalue problem can be solved for the desired number of eigenvalues.

$$\mathbf{K_{uu}}\varphi = \omega_{oc}^2 \mathbf{M_u}\varphi \tag{11}$$

Then using Equation (11) and

$$\mathbf{\Phi^T K_{uu} \Phi} = \mathrm{diag}\left[\, \omega_{oc}^2 1 \;\, \omega_{oc}^2 2 \cdots \omega_{oc}^2 m \,\right] \tag{12}$$

$$\mathbf{\Phi^T M_u \Phi} = \mathbf{I_m} \tag{13}$$

the coupled system of Equation (8) can be reduced to

$$\begin{aligned} &\begin{bmatrix} \mathbf{I_m} & \mathbf{0} \\ \mathbf{0} & \mathbf{M_q} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{r}} \\ \ddot{\mathbf{q}}_\mathbf{e} \end{Bmatrix} + \begin{bmatrix} \mathrm{diag}\,[\mu_m] & \mathbf{0} \\ \mathbf{0} & \mathbf{C_q} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{q}}_\mathbf{e} \end{Bmatrix} \\ &+ \begin{bmatrix} \mathrm{diag}\,[\omega_m^2] & \mathbf{\Phi^T K_{uq}} \\ \mathbf{K_{qu}\Phi} & \mathbf{K_{qq}^*} \end{bmatrix} \begin{Bmatrix} \mathbf{r} \\ \mathbf{q_e} \end{Bmatrix} = \begin{Bmatrix} \mathbf{\Phi^T F_u} \\ \mathbf{F_q} \end{Bmatrix} \end{aligned} \tag{14}$$

where

$$\mathbf{r} = \mathbf{\Phi^{-1} u_e} \tag{15}$$

Thus, the problem has been reduced to a small system composed of only the electrical degrees of freedom and the chosen number of mode shapes. This drastically reduces the number of degrees of freedom and allows for much faster computations once the eigen system is solved for.

OPTIMIZATION TECHNIQUE

Using the developed model, the objective now is to formulate an optimization problem to minimize the vibrational response of a composite plate under a steady state vibrational load. The developed formulation uses a passive electrical damping circuit to control vibration. The circuit is assumed to be composed of linear inductors, resistors and capacitors with values that are determined during the optimization process. Other nonlinear electrical components could be used and optimized, but a corresponding increase in computational effort would be expected. The optimization procedure must also take into account location and orientation of the piezoelectric patches on the plate and determine the location for maximum damping.

The mathematical optimization problem is stated as follows

$$\mathrm{Min}\, f\ (\phi) \tag{16}$$

where

$$\phi = \begin{bmatrix} XC_h \\ YC_h \\ ANG_h \\ R_i \\ L_j \\ C_k \end{bmatrix} \begin{matrix} h = 1 \ldots N_a \\ \\ i = 1 \ldots N_r \\ j = 1 \ldots N_i \\ k = 1 \ldots N_c \end{matrix} \tag{17}$$

subject to the constraints

$$\begin{gathered} R_i \geq 0 \\ L_j \geq 0 \\ C_k \geq 0 \\ \phi_L \leq \phi \leq \phi_U \end{gathered} \tag{18}$$

where f(ϕ) is the objective function representing the peak response of the system for a set of vibrational modes, ϕ is the vector of design variables, N_a is the number of actuators and N_r, N_i, and N_c are the numbers of resistors, inductors, and capacitors to be optimized. The design variables include the *x*- coordinate (*XC*), the *y*-coordinate (*YC*) and the orientation angle (*ANG*) for each actuator and the resistance (*R*), inductance (*L*) and the capacitance values for each electrical component, with upper and lower limits, ϕ_U and ϕ_L, on the electrical values. It is also possible to include ply orientation angles as design variables. Geometric constraints are imposed so that the electrical components maintain positive values in order to represent physical hardware.

It is important to note that the structural design variables (*XC*, *YC*, *ANG*) are all discrete, while the electrical design variables (*R*, *L*, *C*) are all continuous. The electrical design variables are chosen to be continuous since almost any size electrical component can be created by connecting multiple components of various sizes in series or parallel. The *x*- and *y*- coordinates of the piezoelectric patch are chosen as discrete variables based on the finite element model of the plate structure. The plate is meshed with a uniform grid of elements and the piezoelectric patches are required to have locations that align the PZT with the mesh. The size of the piezoelectric patch is chosen a priori, but must be selected as an even multiple of the mesh spacing to ensure that the patch is always aligned with the finite element mesh. This procedure is used to avoid remeshing of the plate during every optimization iteration. The small numerical variations that result from changing the mesh in a finite element analysis can introduce errors that may lead to suboptimal designs. Thus, it is more efficient to use a uniform mesh and restrict the PZTs to discrete locations. The analysis can start with a relatively coarse mesh to determine the general coordinates of the optimum location, and then if further accuracy is desired, the process can be repeated using a more refined mesh and the previous optimum solution as a starting point. The orientation angles are included to allow modeling of inter-digitated electrode (IDE) piezoelectric patches (Rodgers et al., 1996). These actuators are very powerful since they make use of d_{11} actuation and exert an actuation force in a preferential direction.

The optimization formulation is based on the Kreisselmeier–Steinhauser (K–S) function approach. In this technique the multiple objective functions are first transformed into reduced objective functions, which can be expressed as follows

$$\begin{gathered} \hat{f}_i = (F_i(\phi)/F_{i0}) - 1 - g_{\max} \leq 0 \\ i = 1 \ldots \text{number of objective functions} \end{gathered} \tag{19}$$

where F_{i0} represents the original value of the *i*th objective function, F_i is its value based on the current design variables and $g_{\max}$ is the largest value of the original constraint vector. These normalized objective functions are now analogous to constraints and are combined into a single vector, $f_m(\phi)$; $m = 1,2,\ldots,M$, with M being the sum of the number of constraints and the number of objective functions. The constraints and objective functions are then combined into a single composite objective function, $F_{KS}(\phi)$, defined as

$$F_{KS}(\phi) = f_{\max} + \frac{1}{\rho} \ln \sum_{m=1}^{M} e^{\rho(f_m(\phi) - f_{\max})} \tag{20}$$

where $f_{\max}$ is the largest constraint corresponding to the new constraint vector $f_m(\phi)$. The parameter ρ acts as a draw-down factor controlling the distance from the surface of the K–S envelope to the surface of the maximum constraint function.

The optimization algorithm consists of a hybrid scheme that uses simulated annealing to optimize the discrete variables and a continuous search procedure to optimize the continuous variables. Because the K–S function approach reduces the problem to an unconstrained one, any unconstrained gradient based technique can be used for the continuous variables. A modified Newton method is used for the unconstrained. A finite difference scheme is used to calculate the gradient and the Hessian matrix, necessary for determining the search direction. The computational cost associated with the second-order search is mostly overcome by convergence in significantly fewer iterations compared to other first and zero order search techniques. Because simulated annealing is a discrete search method, optimization to the global minimum cannot be guaranteed, although good results can be obtained by adjusting the step sizes and acceptance ratios.

The optimization algorithm begins with the simulated annealing procedure and a user defined initial point. The initial point should be a valid solution, preferably not at an extreme value, but the simulated annealing algorithm should be relatively insensitive to the initial point since it utilizes random search directions. The simulated annealing procedure controls the values of the discrete variables only, and attempts to minimize the objective function using a probabilistic approach. Whenever the value of the objective function is requested by the simulated annealing algorithm, the current values of the discrete variables are used to calculate the open circuit eigen values and eigen vectors of the plate. Then an unconstrained search procedure is invoked to minimize the objective function at that point by varying the continuous design variables. Since the continuous variables are the electrical components, the procedure can be described as determining the optimum passive damping circuit for a particular set of PZT locations and orientations. The objective function value returned to the simulated annealing search is the minimum achievable K–S function value determined by the unconstrained search.

When the value of the objective function is required by the unconstrained search, the current values for the continuous variables are used to construct the linear electrical matrices. These are then combined with the reduced structural matrices based on the eigen values and vectors. These equations are then solved to obtain the system response for any disturbance frequency. Next, the frequency response curves are calculated and the maximum response peak is computed at each mode to be minimized. These parameters are then used to calculate the K–S function value.

RESULTS

The developed optimization procedure is demonstrated first through the integrated design of a cantilevered plate with a single actuator connected to a passive shunt circuit. The cantilevered plate has been studied extensively in the literature related to passive damping and therefore is a good example for benchmarking the developed optimization algorithm. This is due to the fact that for all bending modes, the root is subjected to maximum strain, thus making it the optimal location for a piezoelectric actuator to generate the maximum possible polarization energy and thereby controlling vibration.

First a cantilevered plate similar to that studied by Hagood and Von Flowtow (1991) is examined. The plate is assumed to be made of 3.17 mm thick aluminum and the detailed dimensions are shown in Figure 1. A single piezoelectric actuator is placed on one side to induce vibration and a single actuator is placed on the opposite side and is connected to the shunt circuit. The plate is modeled with a 19×3 element finite element mesh and the system is reduced using the first twelve modes. Optimization is performed to minimize the vibratory response associated with the first mode. The design variables include location of the piezoelectric patch and the electrical parameters governing the inductor and the resistor in the passive shunt. In the initial design, the patch is assumed to be at the center of the plate. Results are presented for two configurations, one with the inductor and resistor in series and the other in parallel.

The frequency response curves before and after optimization are shown in Figure 2. It can be seen that the optimization algorithm is able to reduce the peak response of the first mode by over an order of magnitude in both cases. The optimum location for the piezoelectric patch is determined by the algorithm to be

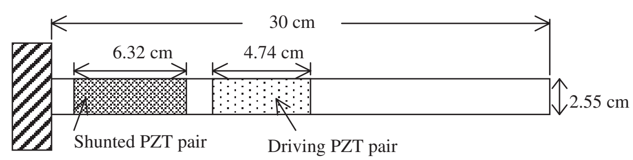

Figure 1. *Configuration for the cantilevered plate prior to optimization.*

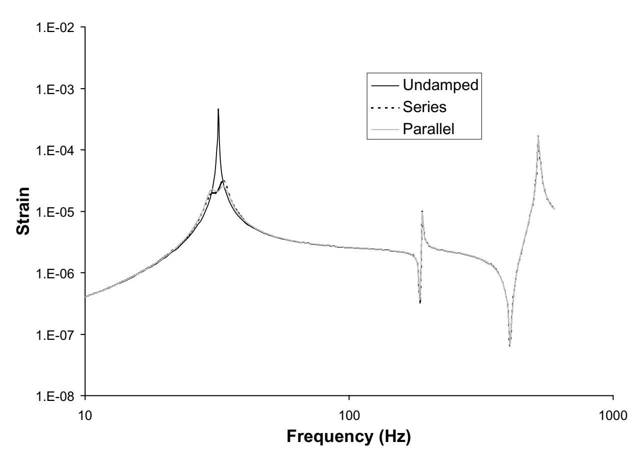

Figure 2. *Frequency response curves for a cantilever plate with and without passive damping circuits.*

at the root of the plate, as was expected for a cantilevered plate. The optimum values calculated for the inductor and resistor in series are 422.032 H and 14406 Ω, respectively. For the case with the inductor and resistor in parallel, the corresponding values are 417.927 H and 489630 Ω. These values cannot be directly compared with the results from Hagood and Von Flowtow since the patches are of slightly different size and the present work uses only a single actuator for damping as opposed to a pair on opposite faces of the plate.

Next a carbon fiber-epoxy composite plate simply supported on all four sides is considered. This example provides a more interesting design challenge due to the complexity of the mode shapes, which are associated with different locations and orientations of maximum strain. In such cases, it is possible that locations and orientations that are optimal for a particular mode may fall on the inflection lines where strain is minimal for the other modes. In such a case the piezoelectric actuator would not generate any polarization energy during vibration that excites that particular mode, making passive damping of that mode impossible. The piezoelectric actuators used in this example are Active-Fiber Composite (AFC) (Rodgers et al., 1996) actuators to allow orientation angle of the actuators to be included as design variables. The plate dimensions are 32 cm × 32 cm with 1.6 mm thickness. The initial lay-up for the plate is eight laminae in $[0,90]_{2s}$ configuration. Vibration is induced by a 2 cm × 2 cm piezoelectric actuator located on the backside of the plate. The plate is modeled with a 16 × 16 finite element mesh with a single 4 cm × 4 cm AFC actuator located in the center of the plate at zero orientation angle as an initial design.

First the AFC patch is modeled with a parallel shunt circuit containing one inductor and one resistor in parallel. The system is then optimized to reduce the response peak for each of the first four vibrational modes. The frequency response curves associated with the initial and the optimum configurations are presented in Figure 3. In each case the targeted mode is reduced by more than a factor of ten, showing the effectiveness of the developed optimization algorithm in damping each of the individual modes. The optimized location of the piezoelectric patch, for the first mode, is the corner of the plate, as illustrated in Figure 4. The optimized orientation angle for the AFC patch is 45° and the

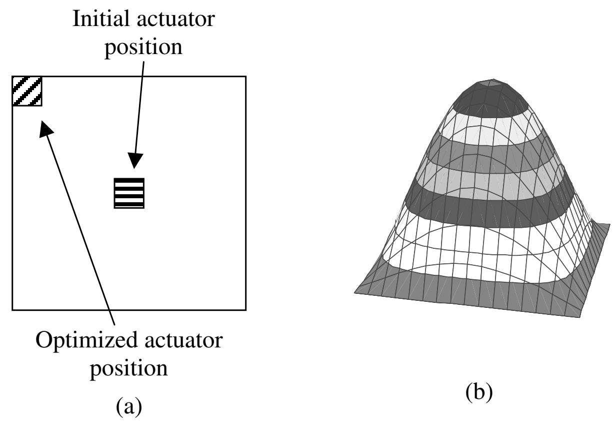

Figure 4. Optimized actuator location, (a), for passive damping of the first vibration mode, (b).

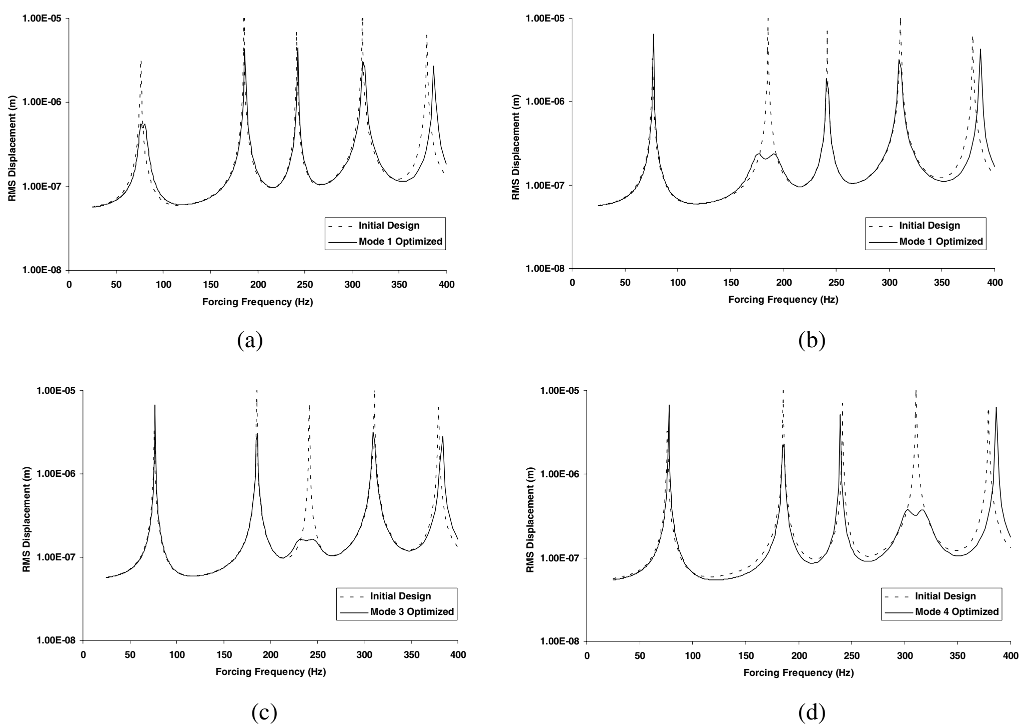

Figure 3. Frequency response curves for the optimal passive damping designs for the first (a), second (b), third (c), and fourth (d) modes.

inductor and resistor assumes values of 110.645 H and 558307 Ω, respectively. The reason the optimal location is in the corner of the plate is because for this mode there is a large bending-shear strain in each of the corners. For the corner shown in Figure 4, this bending-shear strain creates large principle stresses in the AFC patch, which are tensile in the 45° direction and compressive in the −45° direction (or vice versa depending on the out-of-plane deformation). Unlike conventional PZT patches, which expand equally in both directions within the plane of the material, AFC actuators contract in the transverse direction while expanding along the fiber direction. This makes the corner a more optimal location for the patch, as opposed to the center of the plate, where the principle stresses are both compressive (or tensile). For the second and third modes, the optimal location is predicted to be a quarter of the way inwards from the center of one side. These locations coincide with the center of the maximum out-of-plane deflection as shown in Figures 5 and 6. For the second mode the optimal actuator angle is 90°, and the inductor and resistor have values of 20.8077 H and 177846 Ω, respectively. For the third mode the optimal actuator angle is 0°, and the inductor and resistor have values of

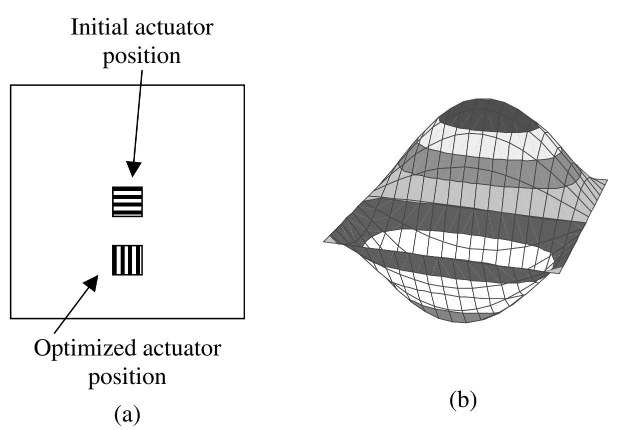

Figure 5. *Optimized actuator location, (a), for passive damping of the second vibration mode, (b).*

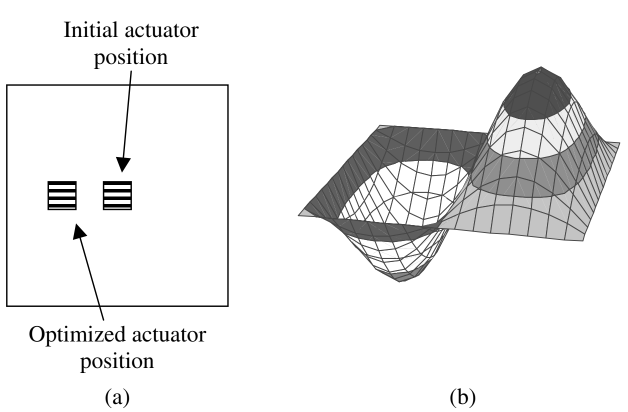

Figure 6. *Optimized actuator location, (a), for passive damping of the third vibration mode, (b).*

12.2968 H and 138118 Ω, respectively. The optimal location for the fourth mode predicted by the optimization algorithm is shown in Figure 7. The optimal angle for the actuator is 60°, and the inductor and resistor have values of 7.2863 H and 192467 Ω, respectively. This location is not intuitively obvious to be the optimal location for the fourth mode. However, convergence to this solution was obtained irrespective of the initial design. This particular case further illustrates the necessity of using formal optimization techniques in the design of structures with complex mode shapes.

The optimum orientation angles of the AFC, for each case studied, are different. This is because, in each case, the piezoelectric fibers of the AFC actuators align with the direction of maximum strain. The results obtained demonstrate the benefit of including both structural and electrical parameters simultaneously in the optimization procedure. In the numerical examples shown, optimal designs are obtained by not only varying the parameters of the passive shunt circuit, but by also tailoring the mechanical stiffness of the piezoelectric patch. The location of the piezoelectric patch not only affects the passive damping capability, it also changes the dynamic response of the system causing natural frequencies to shift. This may or may not be desirable for a particular problem. In such cases the natural frequencies can be included as additional objective functions using the K–S function approach.

Next the same plate is optimized with two 2 cm × 4 cm AFC actuators. Each actuator is connected to a single parallel shunt circuit. In this example the objective is the minimization of the first three vibrational modes. This method is very different from the multimode damping circuit proposed by Wu (1998) which uses three different circuits connected to a single piezoelectric actuator in order to damp the three modes. The method developed by Wu would be less effective in this case since the optimal locations for each mode are different and the use of a single actuator would require compromises in the placement on the plate. The optimized results of the

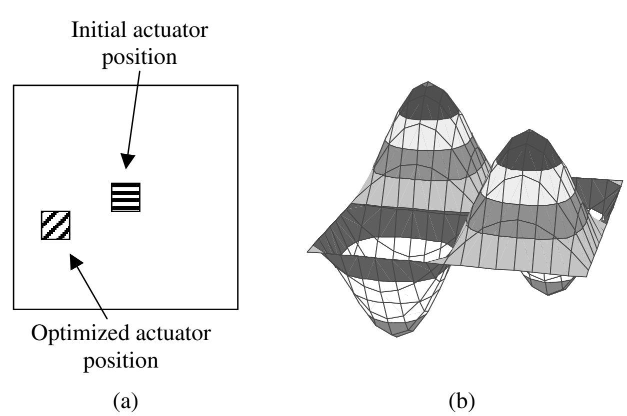

Figure 7. *Optimized actuator location, (a), for passive damping of the fourth vibration mode, (b).*

circuit are shown in Figures 8 and 9. The piezoelectric patches move to locations corresponding to 5 cm × 12 cm and 15 cm × 22 cm, with orientations of 0 and 75° respectively, as seen in Figure 8. The inductor and resistor assume values of 24.2490 H and 620624 Ω for the first actuator and 41.5787 H and 531028 Ω for the second actuator, after optimization. The resulting frequency response curve is presented in Figure 9. It can be seen that this damping circuit affects only the second and third vibration modes. This is because circuits with a single inductor and a single resistor are effective over a narrow range of frequencies. Thus, in this case, two actuators with single circuits cannot damp out more than two of the first three modes. The K–S function definition used in this example caused the solution to favor the second and third modes, since the reduction in peak response was slightly larger than that which could be achieved by damping the first mode. If damping of the first mode was a priority, a weighting parameter can be added to Equation 20 to favor certain modes. The optimal values of the design variables also roughly correspond to the locations and angles predicted for the second and third modes presented in the single actuator case.

In the final case studied, the effect of composite tailoring is investigated by including the ply orientations of the plate as additional design variables. The same plate with two 2 cm × 4 cm AFC actuators is considered. The composite laminae are allowed to be oriented at even multiples of 15°, but symmetry is enforced to reduce the number of design variables. Thus, only four additional design variables are introduced. The optimized results are shown in Figures 10 and 11. The piezoelectric patches move to locations corresponding to 15 cm × 10 cm and 21 cm × 12 cm, with orientations of 75 and 15° respectively, as seen in Figure 10. The inductor and resistor have values of 38.828 H and 559984 Ω for the first actuator and 211.850 H and 230981 Ω for the second actuator. The final ply lay-up for the composite laminate is $[-15°, 75°, 15°, -15°]_s$. The resulting frequency response curve is shown in Figure 11 along

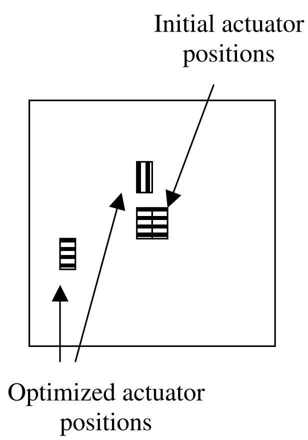

Figure 8. *Position of the actuators on the simply supported plate before and after optimization for two actuators and three modes.*

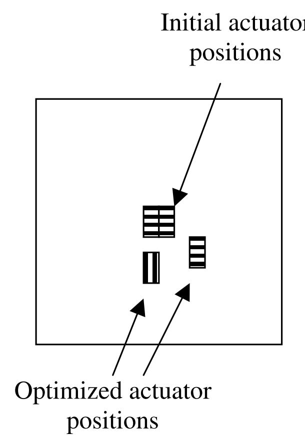

Figure 10. *Position of the actuators on the simply supported plate before and after optimization for two actuators and three modes, including optimization of composite ply orientation.*

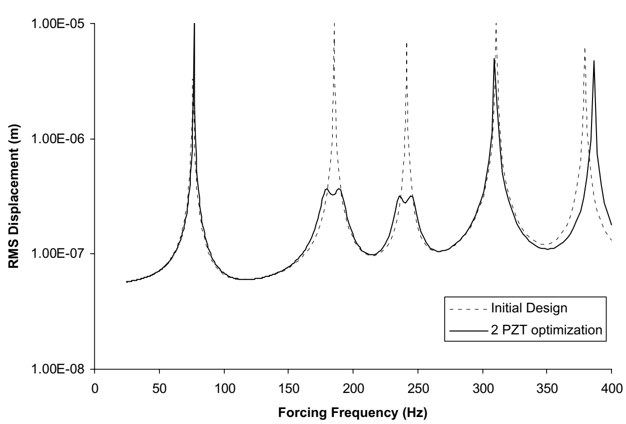

Figure 9. *Frequency response curves for a simply supported plate both undamped and with two actuators.*

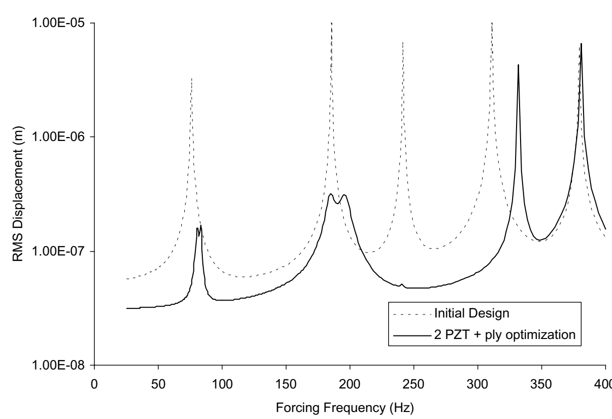

Figure 11. *Frequency response curves for the undamped initial design of a simply supported plate and the optimal design including two actuators and optimized ply orientations.*

with the response curve for the initial cross-ply design. It can be seen that the inclusion of the ply angles are effective in reducing the overall response across the frequency spectrum thus aiding the passive damping actuators which are effective at specific frequencies.

It should be noted that the inductance values predicted in the examples above are quite large and beyond the capabilities of most traditional coil inductors. Others studying passive damping (Wu and Bicos, 1997) have used simulated inductors based on integrated circuits which are capable of creating the effect of large inductors. A drawback to these circuits is that they also create a resistance component. One advantage that the proposed method has is that such behavior can be modeled and included in the optimization process.

The numerical examples presented demonstrate the potential of using MDO techniques for designing integrated adaptive structural systems. For passive damping circuits, optimization determines best possible values of both structural and electrical components and can be used to design systems for damping of multiple modes. But greater value lies in the potential of designing systems with synergistic characteristics, where careful combinations of actuators and circuits provide significant damping with little or no power consumption. Also, this technique can model the behavior of real electrical components as opposed to the ideal components considered thus far. Factors, such as the internal resistance of inductors, can be included in the model without having to reformulate the optimization algorithm.

CONCLUDING REMARKS

A smart structural model has been developed to analytically determine the response of arbitrary structures with piezoelectric materials and attached electrical circuitry. The model simultaneously includes both the structural and the electrical components and can be used to optimally determine both the placement of piezoelectric actuators and parameters describing associated electrical components in a passively damped structure. A robust multiobjective optimization procedure was developed to design the passive system for simultaneous damping of several critical modes of interest. The ability to minimize the frequency response of multiple modes was accomplished by using the Kreisselmeier–Steinhauser function approach. The Kreisselmeier–Steinhauser function approach allows the multiple and conflicting design objectives and constraints to be combined into a single unconstrained function. Since the optimization problem involves both continuous design variables, such as electrical component values, and discrete design variables, such as piezoelectric actuator placement and orientation, a hybrid optimization technique was used. Based on the results obtained during optimization, the following observations were made

- The developed method is capable of optimizing passive damping parameters in various electrical configurations, as demonstrated by optimization of both parallel and series configurations.
- The optimized locations of the piezoelectric actuators correspond to the locations of maximum strain for the mode being optimized, and the fibers of Active Fiber Composites are oriented so that the fibers are parallel to the principle strain.
- Reductions in the frequency response were a result of tuning the circuit to the particular modes as well as overall reductions in response obtained by tailoring the stiffness of the structure.

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REFERENCES

Agnes, G.S. 1994. "Active/Passive Piezoelectric Vibration Suppression," In: *Proc. of the Int. Soc. for Optical Eng.*, Vol. 2193, pp. 24–34.

Belegundu, A.D. and Chandrupatla, T.R. 1999. *Optimization Concepts and Applications in Engineering*, Prentice Hall, Upper Saddle River, New Jersey.

Hagood, N.W. and Von Flotow, A. 1991. "Damping of Structural Vibrations with Piezoelectric Materials and Passive Electrical Networks," *J. of Sound and Vibration*, 146(2):243–268.

Hagood, N.W. and Crawley, E.F. 1991. "Experimental Investigation of Passive Enhancement of Damping for Space Structures," *J. of Guidance, Control and Dynamics*, 14(6):1100–1109.

Kahn, S.P. and Wang, K.W. 1994. "Structural Vibration Controls Via Piezoelectric Materials With Active-Passive Hybrid Networks," In: *Proc. of ASME IMECE*, Vol. DE-75, pp. 187–194.

Rajadas, J.N., Jury, R.A. and Chattopadhyay, A. 2000. "Enhanced Multiobjective Optimization Technique For Multidisciplinary Design," *Eng. Opt.*, 33:113–133.

Rodgers, J.P., Bent, A.A. and Hagood, N.W. 1996. "Characterization of Interdigitated Electrode Piezoelectric Fiber Composites Under High Electrical and Mechanical Loading," In: *Proc. of the Int. Soc. for Optical Eng.*, Vol. 2717, pp. 642–59.

Seeley, C.E., Chattopadhyay, A. and Brei, D. 1996. "Development of a Polymeric Piezoelectric C-Block Actuator Using a Hybrid Optimization Procedure," *AIAA J.*, 34(1):123–128.

Sethi, S.S. and Striz, A.G. 1997. "On Using the Kreisselmeier-Steinhauser Function in Simultaneous Analysis and Design," In: *Proc. of the 38th AIAA/ASME/ASCE Structures, Structural Dynamics and Materials Conf.*, pp. 1357–1365.

Thornburgh, R.P. and Chattopadhyay, A. 2002. "Simultaneous Modeling of Mechanical and Electrical Response of Smart Composite Structures," *AIAA J.*, 40(8):1603–1610.

Tiersten, H.F. 1967. "Hamilton's Principle for Linear Piezoelectric Media," *IEEE Proceedings*, 55(8), 1523–1524.

Tsai, M.S. and Wang, K.W. 1996. "Control of a Ring Structure With Multiple Active-Passive Hybrid Networks," *J. of Smart Materials and Structures*, 5: 695–703.

Tsai, M.S. and Wang, K.W. 1999. "On the Structural Damping Characteristics of Active Piezoelectric Actuators With Passive Shunt," *J. of Sound and Vibration*, 221(1):1–22.

Wrenn, G.A. 1989. "An Indirect Method For Numerical Optimization Using the Kreisselmeier-Steinhauser Function," *NASA Contractor Report 4220*.

Wu, S.Y. 1996. "Piezoelectric Shunts with a Parallel R-L Circuit for Structural Damping and Vibration Control," In: *Proc. of the Int. Soc. for Optical Eng.*, Vol. 2720, pp. 259–269.

Wu, S.Y. 1998. "Method for Multiple Mode Shunt Damping of Structural Vibration Using a Single PZT Transducer," In: *Proc. of the Int. Soc. for Optical Eng.*, 3327: 112–122.

Wu, S.Y. and Bicos, A.S. 1997. "Structural Vibration Damping Experiments Using Improved Piezoelectric Shunts," In: *Proc. of the Int. Soc. for Optical Eng.*, Vol. 3045, pp. 40–50.