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DESIGN SENSITIVITY AND OPTIMIZATION OF COMPOSITE CYLINDERS

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Abstract—Composite cylinders are often used in both rotary- and fixed-wing applications where low weight and high strength are important design issues. This paper, the first of a two-phase study, addresses the failure of such cylinders, under axial compressive loading, using design optimization and sensitivity analysis procedures. Thin-walled cylinders made of different types of symmetric orthotropic laminates and several wall thicknesses are examined. Formal optimization techniques are used and the diameter and individual ply orientations are varied to maximize the critical buckling load of the cylinder. Constraints are imposed on the longitudinal, normal and in-plane shear stresses of each ply. The optimization is performed using the nonlinear programming method of feasible directions. A two-point exponential approximation method is also used to reduce computational effort. Results are presented for Graphite/Epoxy, Glass/Epoxy and Kevlar/Epoxy composite cylinders with symmetric ply arrangements.

NOMENCLATURE

$[A]$	extensional stiffness matrix
$[B]$	coupling stiffness matrix
$[D]$	bending stiffness matrix
E_1	longitudinal elastic modulus of composite
E_2	transverse elastic modulus of composite
$F(\phi)$	objective function
G_{12}	in-plane shear elastic modulus of composite
L	length of cylinder
$\{M\}$	resultant moments on laminate
$\{N\}$	resultant forces on laminate
N_x	buckling load
$N_{x_{cr}}$	critical buckling load
$[Q']$	off-axis stiffness matrix
R	inner radius of cylinder
$[T]$	transformation matrix
V_F	volume fraction of fibers in composite
$g_j(\phi)$	constraint functions (ply stresses $\sigma_1, \sigma_2, \sigma_{12}$)
$\{k\}$	laminate curvatures
m	number of buckle half-waves in the axial direction
n	number of buckle waves in the circumferential direction
$\{s\}$	off-axis stresses
t	cylinder wall thickness
z	distance from laminate mid-plane to ply surfaces
α, β	buckling parameters
$\{\varepsilon^0\}$	laminate mid-plane strains
γ	buckling load correlation factor
ν_{12}	major Poisson ratio
ϕ	design variable vector
ρ	density of composite material
σ_{1T}	longitudinal tensile strength
σ_{1C}	longitudinal compressive strength
σ_{2T}	transverse tensile strength
σ_{2C}	transverse compressive strength
σ_{12}	in-plane shear strength
σ_T	tensile strength bound
σ_C	compressive strength bound
$\{\sigma\}$	material-axis stresses
θ	ply angle.

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INTRODUCTION

Structural optimization has become an efficient tool and is being used to expedite design in several disciplines. An extensive amount of work has been done in developing design optimization procedures to bring the state-of-the-art to a very high level (Schmit, 1981; Vanderplaats, 1982). In the past, conventional design methods mainly used the designer's experience and trial-and-error methods. However, today the availability of sophisticated computing resources makes it possible to use design optimization procedures at various stages of design.

A significant amount of work has been done in the application of optimization procedures for mechanical design. However, a great majority of this work has been limited to isotropic materials where the design variables are generally size, shape and topology. With the increasing demand for the use of fiber-reinforced composites, it is now necessary to consider design parameters related to material properties, either at the ply level or at the laminate level. Due to the importance of the problem, there has been some effort at using structural optimization procedures for design with composites in recent years (Kicher and Chao, 1971; Vanderplaats and Weisshaar, 1989; Zimmermann, 1989; Fukunaga and Vanderplaats, 1991; Pederson, 1991; Gürdal and Haftka, 1991). Typically, structures were designed for minimum weight using ply orientations, thicknesses and sometimes number of plies as design variables. Vanderplaats and Weisshaar (1989) addressed the optimization of membrane panels, both unstiffened and stiffened, with constraints on strains and frequencies. The design of an aeroelastically tailored wing was also discussed in this paper. Most recently, Pederson (1991) addressed the problems of optimal ply orientation of a uniform cantilever beam and a bending-loaded knee. He also addressed the problem of optimal shape design using orthotropic material. Gürdal and Haftka (1991) used integer programming and other novel optimization techniques such as the Genetic Algorithm to address design problems where ply orientations cannot be varied and ply stacking sequences are the only design variables. In the design of composite cylinders, the problem of minimum weight has been addressed by Kicher and Chao (1971). The optimization of axially compressed composite cylindrical shells for minimum weight was also addressed by Zimmermann (1989), where ply orientations were used as design variables with constraints on the buckling load. Since the weight of a structure is independent of the ply orientations, in the optimization problem, the objective function is not a function of the design variables whereas the constraints are. This poses a considerable amount of problems as any feasible solution is a possible optimum. Fukunaga and Vanderplaats (1991) used lamination parameters as design variables to optimize the buckling loads of cylinders under combined loadings. These parameters are useful for optimization purposes but are not readily available or easily used in design.

The present paper is the first part of a two-phase study on the buckling and post-buckling of composite cylinders. The initial phase, addressed in this paper, is to use design optimization procedures to maximize the critical buckling load of composite cylinders, with constraints on ply stresses, and to study the sensitivity of this load with respect to constituent material and geometric properties of the cylinders. In comparison to previous research in this field, more practical and realistic design variables and constraints are used to formulate the optimization problem. The sensitivity of the buckling load with respect to material constitutive properties and geometry is also examined.

PROBLEM DESCRIPTION

This paper addresses the use of sensitivity analysis and formal optimization techniques for the design of axially compressed thin-walled composite cylinders. The goal is to prolong the onset of failure due to first buckling by using geometric and material design variables. Since failure occurs if at least one unidirectional layer looses its strength by exceeding the allowable stress limit, constraints are imposed on the longitudinal, normal and in-plane shear stresses at each ply level. Cylinders made of three different types of orthotropic symmetric laminates and varying wall thicknesses are considered. By varying the cylinder radius and individual ply orientations, specific combinations are obtained that increase the buckling load, with respect to a baseline or reference cylinder. A structural analysis procedure, based on classical laminate theory, is used for the analysis and a nonlinear programming technique combined with an approximate analysis method is used for the optimization.

STRUCTURAL MODEL

A geometric illustration of a typical orthotropic composite cylinder and laminate considered in this study is presented in Figs 1 and 2. Symmetric laminates are used to reduce the number of design variables. The initial or reference cylinders all have a length, L, of 50 in., an inner radius, R, of 10 in. ($L/R = 5$) and ply orientations of alternating $\pm 30°$ (e.g. a 6-ply cylinder has a reference orientation of $[+30°/-30°/+30°]_s$). A $\pm 30°$ lay-up scheme was chosen because it is a typical configuration used in industry. The total number of plies used to make up the wall thickness, t, is varied in each specimen. The plies are numbered starting from 1 through to the final ply in the laminate, with the first ply being the outer most one (Fig. 1). A value of 0.01 in. is used for the thickness of each ply. Five cylinders of Graphite/Epoxy (Gr/Ep), Glass/Epoxy (Gl/Ep) and Kevlar/Epoxy (K/Ep) with two, four, six, eight and 10 plies are optimized for buckling. The properties of these constituent materials (Vinson and Sierakowski, 1987) are presented in Table 1.

ANALYSIS AND OPTIMIZATION

In this section, a brief description of the structural analysis procedure is provided, followed by a formulation of the optimization problem. The implementation of the optimization procedure is described thereafter.

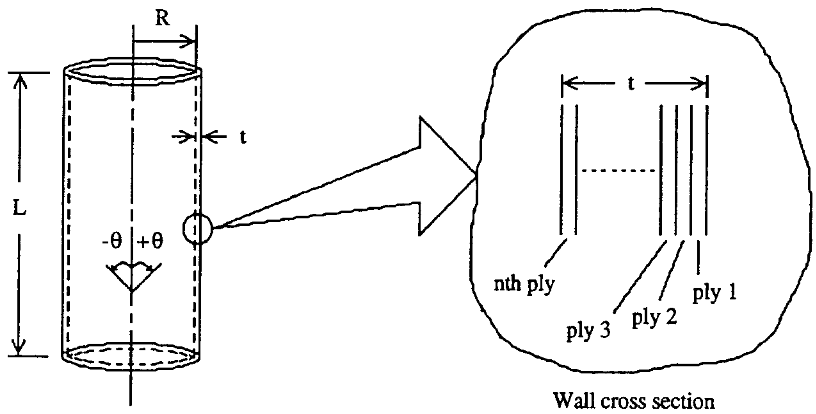

Fig. 1. Typical composite cylinder.

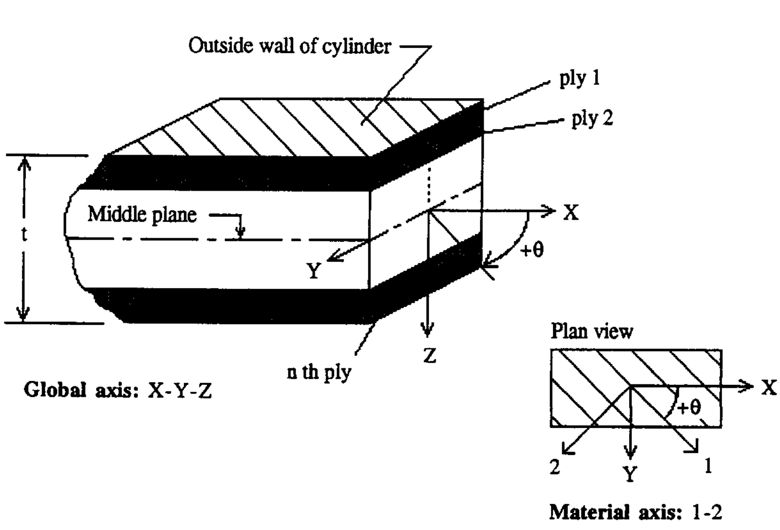

Fig. 2. Axis orientation and geometric representation of laminates.

Table 1. Composite material properties

	Gr/Ep	Gl/Ep	K/Ep
E_1 (psi)	22,200,000	8,800,000	11,020,000
E_2 (psi)	1,580,000	3,600,000	798,000
G_{12} (psi)	810,000	1,740,000	334,000
ν_{12}	0.3	0.23	0.34
σ_{1T} (psi)	100,000	187,000	203,050
σ_{1C} (psi)	110,000	119,000	34,080
σ_{2T} (psi)	4,000	6,670	1 ,740
σ_{2C} (psi)	13,900	25,300	7,690
σ_{12} (psi)	9,000	6,500	4,930
ρ (lb/in^3)	0.058	0.073	0.050
V_F (%)	70	72	60

Laminate analysis

Classical laminate theory (Vinson and Chou, 1975; Agarwal and Broutman, 1980; Vinson and Sierakowski, 1987) is used to analyze the individual material-axis ply stresses of the orthotropic cylinders. The following are the constitutive relations provided by this theory that relate the ply strains and curvatures to the resultant forces and moments:

$$\{N\} = [A]\{\varepsilon^0\} + [B]\{k\} \tag{1}$$

and

$$\{M\} = [B]\{\varepsilon^0\} + [D]\{k\}. \tag{2}$$

The form of the above general constitutive relations can be extended to thin-walled cylinders. The relation is simplified in the case of axial compressive loading, with all the elements of the matrix $\{M\}$ being zero and only one non-zero element, N_x (the compressive stress), in matrix $\{N\}$. These equations are solved for mid-plane strains and curvatures. The corresponding individual ply stresses are evaluated as follows:

$$\{s\} = [Q']\{\varepsilon^0\} + z[Q']\{k\} \tag{3}$$

$$\{\sigma\} = [T]\{s\} \tag{4}$$

where the positive direction of the z-axis points from the laminate (cylinder wall) mid-plane toward the center of the cylinder (Fig. 2).

Buckling analysis

The critical buckling load, the subject of this study, represents the value of N_x which is a minimum with respect to variations in buckling modes. An accurate analysis of this load is important since the accuracy of individual ply stress calculations depends on this. Theoretical results presented by two NACA studies in the early 1950s (Stein and Mayers, 1951, 1952) provide insight into the buckling of cylindrical shells of sandwich construction. These studies present the necessary governing differential equations, based on small-deflection theory, needed to analyze the buckling of sandwich cylinders. The above studies were based on the classical plate theory and therefore out-of-plane terms were neglected. In reality, however, this is not the case for general symmetric laminates. The analysis was later modified and a correlation factor, γ, was introduced to reduce the discrepancy between the results obtained from classical theory and experiments (Anon, 1968). Results of buckling loads obtained in this paper are based on the form of this equation as presented in Vinson and Sierakowski (1987). This equation is stated below:

$$\frac{N_x L^2}{\pi^2 D_{11}} = m^2\left(1 + 2\frac{D_{12} + 2D_{66}}{D_{11}}\beta^2 + \frac{D_{22}}{D_{11}}\beta^4\right) + \frac{\gamma^2 L^4}{\pi^4 m^2 D_{11} R^2} \times \frac{A_{11}A_{22} - A_{12}^2}{A_{11} + ((A_{11}A_{22} - A_{12}^2)/A_{66} - 2A_{12})\beta^2 + A_{22}\beta^4} \tag{5}$$

where

$$\beta = \frac{nL}{\pi Rm} \tag{6}$$

$$\gamma = 1.0 - 0.901\,(1 - e^{-\alpha}) \tag{7}$$

and

$$\alpha = \frac{1}{29.8}\left[\frac{R}{\sqrt[4]{D_{11}D_{22}/A_{11}A_{22}}}\right]^{1/2}. \tag{8}$$

The critical buckling load, $N_{x_{cr}}$, is obtained by varying m and n (the number of buckle waves in the axial and circumferential directions, respectively) until a combination is found that produces the lowest load, N_x. The boundary conditions for the case of a simply-supported cylinder are assumed in the analysis.

Optimization formulation

The objective is to maximize the critical buckling load, $N_{x_{cr}}$, with constraints on the material axis ply stresses, $\sigma_1, \sigma_2, \sigma_{12}$ (Fig. 2). The cylinder inner radius (R) and ply orientations (θ_i, $i = 1, \ldots, n$) are used as design variables. Upper-bound constraints are imposed on the stresses based on the failure strength of the particular composite material used for the cylinder. In a standard optimization formulation, the problem is that of minimizing an objective function. Since the problem addressed in this paper is one of maximization of the buckling load, a new objective function is defined as the negative of the buckling load. This then allows the application of the techniques of minimization to produce a maximum. The formal optimization problem can be stated as follows:

Minimize:

$F(\phi)$ (objective function)

where

$F = -N_{x_{cr}}$,

subject to:

$g_j(\phi) \le 0$ $\quad j = 1, \ldots, \text{NCON}$ (constraints)

$\phi_{i_L} \le \phi_i \le \phi_{i_U}$ $\quad i = 1, \ldots, \text{NDV}$ (side constraints)

where NDV is the number of design variables and NCON is the total number of constraints. Side constraints are imposed on the design variables to avoid unrealistic designs. The cylinder inner radius is allowed to change by $\pm 25\%$ and plies are allowed to vary between $\pm 90°$. Note that the values of NDV and NCON are different for cylinders with different wall thicknesses. Since symmetric laminates are used, only the orientations of half the plies are used as design variables and the stresses in half the plies are constrained.

Optimization implementation

The optimization process is initiated by defining all the necessary preassigned parameters (e.g. cylinder length and wall thickness) for the problem. Next, the design variables are initialized and the structural analysis is performed. The structural analysis consists of the calculation of all the necessary classical laminate constitutive properties. The objective function and the constraints are then evaluated, followed by a sensitivity analysis. During optimization, changes in the cylinder's parameters can change the critical buckling mode. Therefore, to find $N_{x_{cr}}$ (objective function) the values of m and n [eqn (5)] are varied in combinations from 1 to 10 and 0 to 10, respectively, and the lowest value of N_x is used as $N_{x_{cr}}$ for a particular cycle. The optimizer consists of the nonlinear programming technique (the method of feasible directions) as implemented in the computer-code CONMIN (Vanderplaats, 1973) and a two-point exponential approximation method (Fadel *et al.*, 1990). The two-point method employs an exponential function, which utilizes previous analytical information, in a first-order Taylor-series relation. The exponent helps to improve the fit of the series expansion. The approximate analysis is used to reduce the computational effort involved in using the exact analysis for several evaluations of the objective function and constraints necessary within CONMIN. To reduce possible errors in the approximations, move limits, defined as the maximum fractional change of a design variable value, are imposed as upper and lower bounds on the design variables, ϕ_i. Convergence is based upon the objective function value over three consecutive cycles, where a cycle comprises a complete analysis and optimization. A convergence tolerance of 0.005 is used.

RESULTS AND DISCUSSION

Results obtained by using the above optimization procedure are presented in this section. The results of the optimization are compared against a reference design. The material and number of plies are varied to investigate their sensitivity to the buckling load. A total of 15 cylinders was analyzed; five cylinders of Graphite/Epoxy, Glass/Epoxy and Kevlar/Epoxy with two, four, six, eight and 10 plies, respectively. In each case, optimum configurations for maximum buckling load are obtained with 6–21 cycles. A variable move limit procedure is employed in which a larger move limit is used initially to accelerate the process and a tighter limit is used as the optimizer approaches a minimum. Move limits of the order of 0.1–0.01 are used.

The results of the optimization are summarized in Table 2 and Figs 3–5. Table 2 presents the buckling loads of both the reference and the optimum cyclinders along with the associated buckling mode shapes, i.e. the parameters m and n. The Gr/Ep cylinders are able to support the highest overall buckling loads in both the reference and the optimum cylinders, followed by the Gl/Ep and K/Ep cylinders. The values of the buckling-mode shape parameters change from reference to optimum but display no pattern in relation to the number of plies or buckling loads. However, it is important to note that it is essential to evaluate the values of the parameters at each step of the design optimization since the mode of buckling changes with changes in the design variables. The increases in buckling loads of the reference and optimum cylinders are presented in Figs 3–5. Figure 3 shows an increase of the buckling loads, from reference to optimum, for the Gr/Ep cylinders. A maximum increase (177%) occurs in the 4-ply cylinder, and the 2-ply cylinder has the lowest overall increase (55%). Similar trends are exhibited by the K/Ep cylinders (Fig. 4). The overall increase in buckling strength is the lowest for the Gl/Ep cylinders. In this case, the 4-ply cylinder shows the maximum increase in buckling load (66%) and the 10-ply cylinder yields the lowest increase (49%), as shown in Fig. 5. However, no conclusion can be drawn regarding this nonlinear change of optimum

Table 2. Critical buckling loads for the reference and optimum cylinders

	Number of plies	Reference		Optimum	
		Nx_{cr} (lb/in)	(m,n)	Nx_{cr} (lb/in)	(m,n)
Gr/EP	2	41.7	1,4	64.6	3,7
	4	222.4	1,4	616.5	8,7
	6	621.8	1,4	1,523.3	10,7
	8	1,305.2	2,5	3,024.3	4,5
	10	1,971.7	10,0	4,836.7	3,4
Gl/Ep	2	39.2	1,4	62.2	2,5
	4	216.1	2,5	359.3	8,7
	6	580.1	2,5	910.2	4,5
	8	1,108.0	1,3	1,675.5	7,6
	10	1,793.4	1,3	2,668.0	4,5
K/Ep	2	19.7	1,4	31.7	1,4
	4	106.2	1,4	303.5	2,4
	6	299.1	1,4	747.2	2,4
	8	600.6	10,0	1416.6	4,5
	10	908.3	10,0	2409.8	10,6

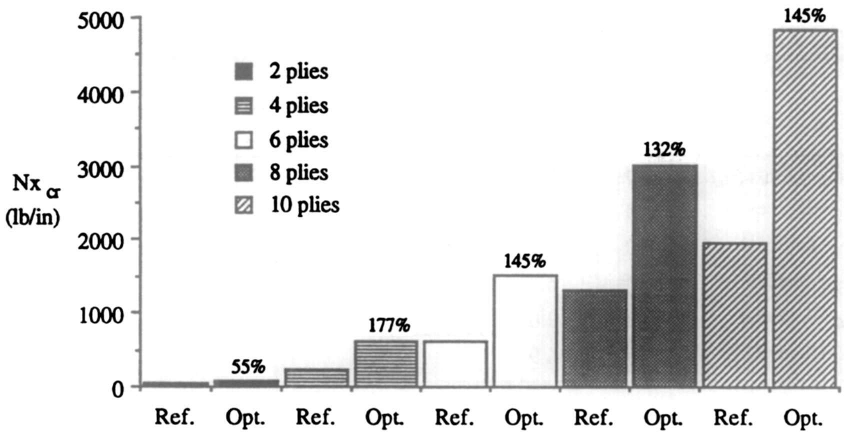

Fig. 3. Comparison of buckling loads for Gr/Ep cylinders.

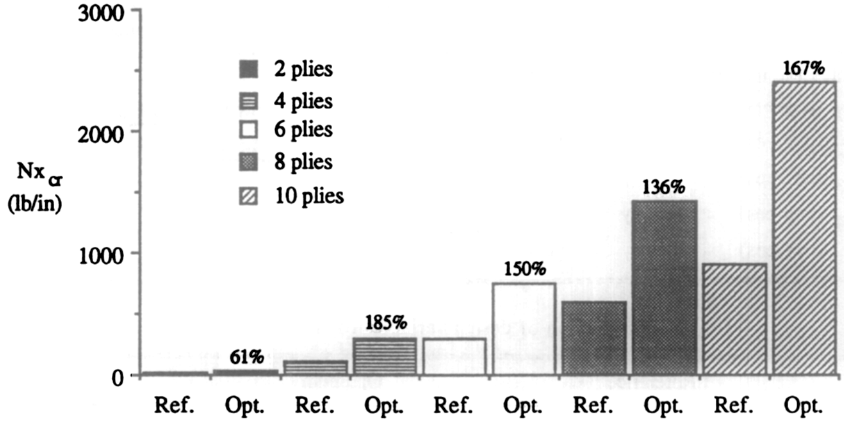

Fig. 4. Comparison of buckling loads for K/Ep cylinders.

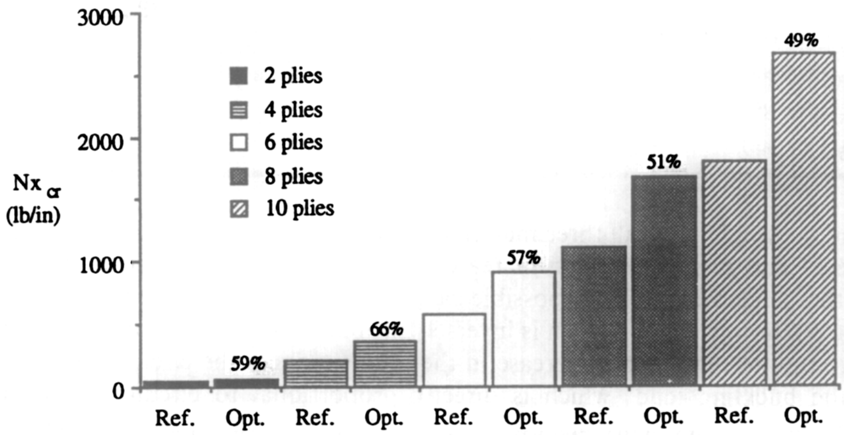

Fig. 5. Comparison of buckling loads for Gl/Ep cylinders.

buckling load with changes in the number of plies, due to the possible existence of local minimums. The buckling loads of both the reference and the optimum cylinders increase, with the number of plies, almost linearly. This is expected, since the wall thickness plays a crucial role in supporting the loads and controlling buckling.

Tables 3–5 present the design variable values for the reference and optimum cylinders. Only half the ply orientations (θ_i) are presented, due to conditions of symmetry. From each of these tables, some interesting trends can be observed regarding the radius and the ply orientations. For all 15 cylinders, the radii decreased from their initial value of 10 in. in the reference cylinders to a value of 7.5 in. (design variable lower bound)

Table 3. Comparison of design variables for Gr/Ep cylinders

	Reference	Optimum				
		10 plies	8 plies	6 plies	4 plies	2 plies
Inner radius (in)	10.0	7.5	7.5	7.5	7.5	7.5
θ_1 (degrees)	30.0	50.9	54.4	53.1	56.9	16.1
θ_2 (degrees)	-30.0	-49.1	-45.4	-19.5	-15.2	
θ_3 (degrees)	30.0	16.9	15.4	15.9		
θ_4 (degrees)	-30.0	-19.0	-15.4			
θ_5 (degrees)	30.0	16.9				

Table 4. Comparison of design variables for Gl/Ep cylinders

	Reference	Optimum				
		10 plies	8 plies	6 plies	4 plies	2 plies
Inner radius (in)	10.0	7.5	7.5	7.5	7.5	7.5
θ_1 (degrees)	30.0	22.5	42.7	52.1	58.5	26.5
θ_2 (degrees)	-30.0	-26.7	-18.5	-22.8	-14.3	
θ_3 (degrees)	30.0	22.5	12.2	18.7		
θ_4 (degrees)	-30.0	-19.3	-12.2			
θ_5 (degrees)	30.0	18.7				

Table 5. Comparison of design variables for K/Ep cylinders

	Reference	Optimum				
		10 plies	8 plies	6 plies	4 plies	2 plies
Inner radius (in)	10.0	7.5	7.5	7.5	7.5	7.5
θ_1 (degrees)	30.0	52.4	54.0	52.8	57.0	16.7
θ_2 (degrees)	-30.0	-60.5	-18.3	-18.3	-15.2	
θ_3 (degrees)	30.0	17.4	22.4	16.5		
θ_4 (degrees)	-30.0	-15.7	-22.4			
θ_5 (degrees)	30.0	26.8				

in the optimum cylinders for all three materials. This changes the slenderness ratio (L/R) from 5 to 6.67 and suggests that an increase in the buckling load is attributable to an increase in the slenderness ratio. A possible cause is the reduction in the parameter β, in the buckling relation [eqn (5)], which is inversely proportional to the slenderness ratio. It is also interesting to note that a decrease in the radius reduces the circumferential area. However, the buckling load, which is directly proportional to circumferencial area, increases for all cylinders. This suggests that the ply orientations play a crucial role in increasing the critical buckling load. Another interesting observation is that by decreasing the radius the surface area and thus weight of the cylinder is reduced. The optimum cylinders therefore support higher buckling loads and are lighter than the respective reference cylinders. Tables 3–5 also indicate that the ply angles closer to the mid-plane of the cylinder wall generally decrease in magnitude (from their reference value) and those nearer the outer surface increase in magnitude.

Table 6 presents the constraint values of the reference and the optimum cylinders for the case of Gr/Ep. The constraints are on the longitudinal, transverse and in-plane shear stresses (σ_1, σ_2, σ_{12}) along the material axis of each ply. Only half the ply stresses for each cylinder are shown, due to conditions of symmetry. In these tables, the upper bounds imposed on each of these stresses, σ_T and σ_C, represent the material strength in tension

Table 6. Constraint values for the reference and optimum Gr/Ep cylinders

		Bounds (psi)		10 plies		8 plies		6 plies		4 plies		2 plies	
		σ_C	σ_T	ref.	opt.	ref.	opt.	ref.	opt.	ref.	opt.	ref.	opt.
1†	σ_1	-110,000	100,000	-19,227	12,612	-18,466	515	-9,283	1,418	-6,292	-5,914	-1,565	-2,982
	σ_2	-13,900	4,000	2,364	-2,724	2,151	-2,110	1,122	-1,041	733	100	-521	-250
	σ_{12}	-9,000	9,000	4,367	5,901	3,468	4,823	2,376	2,916	1,182	3,600	903	864
2†	σ_1	-110,000	100,000	-26,747	-12,991	-18,466	-7,225	-16,246	-36,068	-6,292	-26,144		
	σ_2	-13,900	4,000	2,747	-1,402	2,151	-1,716	1,476	867	733	1,130		
	σ_{12}	-9,000	9,000	-4046	-6,237	-3,468	-4,923	-2,078	-2,048	-1,182	-3,117		
3†	σ_1	-110,000	100,000	-19,227	-78,300	-18,466	-75,262	-9,283	-42,540				
	σ_2	-13,900	4,000	2,364	1,905	2,151	1,747	1,122	1,197				
	σ_{12}	-9,000	9,000	4,367	4,089	3,468	2,105	2,376	1,374				
4†	σ_1	-110,000	100,000	-26,747	-86,861	-18,466	-68,570						
	σ_2	-13,900	4,000	2,747	2,340	2,151	1,407						
	σ_{12}	-9,000	9,000	-4046	-3,215	-3,468	-2,938						
5†	σ_1	-110,000	100,000	-19,227	-78,300								
	σ_2	-13,900	4,000	2,364	1,904								
	σ_{12}	-9,000	9,000	4,367	4,089								

† Ply number.

and compression, respectively. Note that the reference values change as the wall thicknesses are changed. It is obvious from these tables that optimization leads to significant changes in the stress distributions at each ply level. The stresses remain well within the prescribed bounds. However, in some cases the nature of the stresses changes from tensile to compressive and *vice versa* after optimization. For example, in the 6-ply cylinder at ply 1, σ_1 is compressive in the reference cylinder and tensile in the optimum cylinder. The nature of these stresses also varies significantly with changes in the number of plies. For example, at ply 1 of the optimum configuration, σ_2 remains compressive for the 2-, 6-, 8- and 10-ply arrangements, but becomes tensile in the 4-ply arrangement. Similar trends were observed for the Gl/Ep and K/Ep cylinders. By examining Table 6 it is obvious that the dominating stresses in the cylinder walls are the normal and shear stresses. The high values of these stresses can attribute to mechanisms which initiate cylinder buckling.

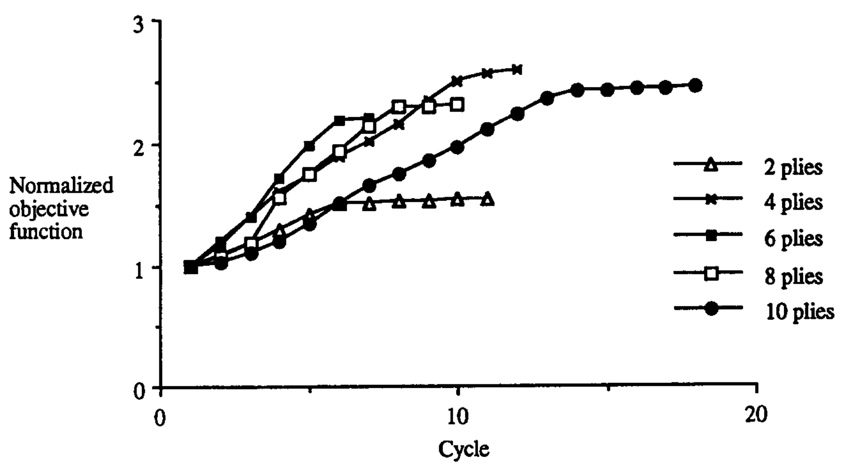

Fig. 6. Objective function iteration histories for Gr/Ep cylinders.

Figures 6–8 present the iteration histories of the objective functions for each of the cylinders. The consistent increase in the objective function values ($-N_{x_{cr}}$), in all cases, is due to the fact that the initial designs are feasible designs, i.e. designs satisfying all the constraints. The smoothest convergence occurs in the case of the Gr/Ep and K/Ep cylinders.

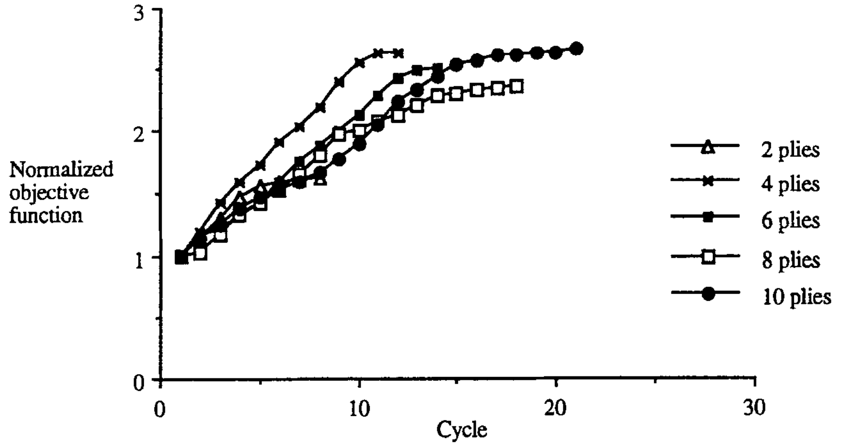

Fig. 7. Objective function iteration histories for K/Ep cylinders.

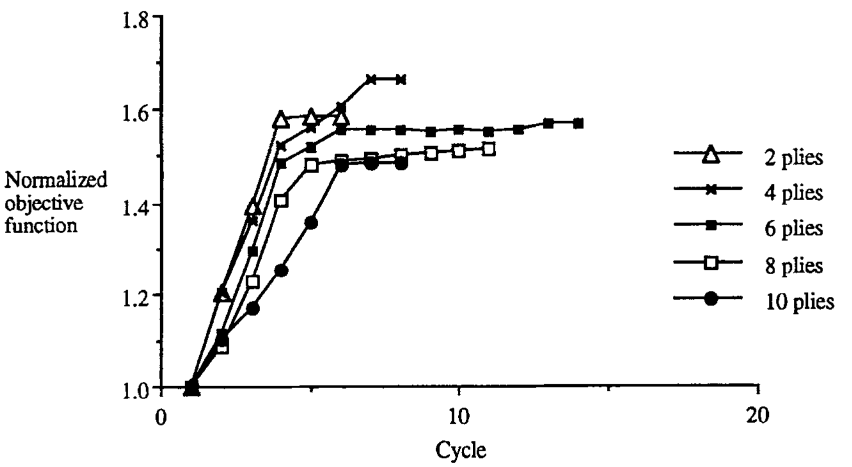

Fig. 8. Objective function iteration histories for Gl/Ep cylinders.

CONCLUDING REMARKS

In this paper, the optimum design of composite cylinders, under axial compressive loading, was addressed for maximizing the load-carrying capability of the cylinders. A sensitivity analysis was performed to study the effect of the total number of plies and the material constituent properties on the buckling load. During optimization, an analysis was performed to evaluate the corresponding buckling load with each design change. The radius of the cylinder and the ply orientations were used as the design variables. Constraints were imposed on the ply longitudinal, normal and in-plane shear stresses. The optimization was performed using the method of feasible directions. A two-point exponential approximation method was used to reduce computational effort. Results were presented for cylinders made of Gr/Ep, Gl/Ep and K/Ep orthotropic laminates with five different wall thicknesses. Optimum configurations for maximum buckling load were obtained within 6–21 cycles. The following are some observations made from this study:

(1) Optimization increased the magnitudes of the critical buckling loads from the reference values. Gr/Ep and K/Ep yield the highest increases.
(2) The mode of buckling (parameters m and n) changes from reference to optimum.
(3) Significant changes occur in the values of the design variables. Reductions are obtained in the cylinder weights, although weight was not used as a constraint.
(4) The magnitudes and the nature of the stresses in each ply change significantly from reference to optimum and with changes in the wall thicknesses. Gr/Ep and K/Ep displayed the most significant changes.
(5) The objective function and the constraint convergences are smoother for the Gr/Ep and K/Ep cylinders.

REFERENCES

Agarwal, B. D. and Broutman, L. J. (1980). *Analysis and Performance of Fiber Composites*. John Wiley, New York.

Anon (1968). Buckling of thin walled circular cylinders. NASA SP-8007.

Fadel, G. M., Riley, M. F. and Barthelemy, J. F. (1990). Two-point exponential approximation method for structural optimization. *Struct. Optimiz.* **2**, 117–124.

Fukunaga, H. and Vanderplaats, G. N. (1991). Stiffness optimization of orthotropic laminated composites using lamination parameters. *AIAA Jl* **29**(2), 641–646.

Gürdal, Z. and Haftka, R. T. (1991). Optimization of composite laminates. NATO/DFG ASI, *Optimization of Large Structural Systems*, Lecture Notes, Vol. 1, pp. 200–215, Berchtesgaden, Germany, Sept. 1991.

Kicher, T. P. and Chao, T. L. (1971). Minimum weight design of stiffened fiber composite cylinders. *J. Aircraft* **8**(7), 562–568.

Pederson, P. (1991). Optimal orientation of anisotropic materials. NATO/DFG ASI, *Optimization of Large Structural Systems*, Lecture Notes, Vol. 1, pp. 168–200, Berchtesgaden, Germany, Sept. 1991.

Schmit, L. A. (1981). Structural synthesis—its genesis and development. *AIAA Jl* **19**(10), 249–1263.

Stein, M. and Mayers, J. (1951). A small-deflection theory for curved sandwich plates. NACA Report 1008.

Stein, M. and Mayers, J. (1952). Compressive buckling of simply supported curved plates and cylinders of sandwich construction. NACA TN 2601.

Vanderplaats, G. N. (1973). *CONMIN—A Program for Constrained Function Minimization*. User's Manual. NASA TMX-62282.

Vanderplaats, G. N. (1982). Structural optimization—past, present and future. *AIAA Jl* **20**, 992–1000.

Vanderplaats, G. N. and Weisshaar, T. A. (1989). Optimum design of composite structures. *J. Numer. Meth. Engng* **19**, 437–448.

Vinson, J. R. and Chou, T. W. (1975). *Composite Materials and Their Use in Structures*. Applied Science Publishers, London.

Vinson, J. R. and Sierakowski, R. L. (1987). *The Behavior of Structures Composed of Composite Materials*. Kluwer, Dordrecht.

Zimmermann, R. (1989). Optimization of axially compressed fiber composite cylindrical shells. *Proc. International Seminar Organized by Deutsche Forschungsanstalt für Luft- und Raumfahrt (DLR)*, Bonn, June 1989.