DAMAGE DETECTION AND VIBRATION CONTROL OF A DELAMINATED SMART COMPOSITE PLATE

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SUMMARY: In this paper, the effects of delamination on the dynamic characteristics of a composite plate are investigated. The refined higher order theory is used to model the smart composite plate in the presence of delaminations. The theory accurately captures the transverse shear deformation through the thickness, which is important in anisotropic composites, particularly in the presence of discrete actuators and sensors and delaminations. Next, the detection of delamination is investigated using the Root Mean Square (RMS) values of the response of the composite plate subject to disturbances. An active control system is designed to minimize the effect of delamination. The pole placement technique is applied to design the closed loop system by utilizing piezoelectric actuators. Numerical results show that the RMS information can be used to find the exact location of the delamination. The controller designed makes the delaminated plate behave like a healthy plate model. The controller also reduces the magnitudes of RMS responses due to disturbance.

KEYWORDS: damage detection, delaminations, active vibration control, smart composite modeling.

INTRODUCTION

Imperfections, such as delaminations which may occur during the life of a structure can greatly change its characteristics and must be carefully investigated. Existence of delaminations causes reductions in natural frequencies and increases in vibratory damping. Dynamic response measurements are a very attractive form of damage detection tests since they can be made at a single point on the component and are independent of the position chosen. Therefore, accurate modeling of the structural dynamic characteristics of delaminated composites is an important issue. A significant amount of research has been performed in modeling defects such as delamination in composites. Although three dimensional approaches [1,2] are more accurate than two dimensional theories [3-6], their implementation can be very expensive for practical applications. The layer-wise approach [7] is an alternative since it is capable of modeling displacement discontinuities. However, the computational effort increases with the number of plies. Recently, a refined higher order theory, developed by Chattopadhyay and Gu [8,9], was shown to be both accurate and efficient for modeling delamination in composite plates and shells of moderately thick construction. This theory has also been shown to agree well with both elasticity solutions [10] and experimental results [11]. Relatively little attention has been paid to detailed modeling issues associated with adaptive composite structures, with surface bonded/embedded piezoelectric actuators and sensors, in the presence of defects. In most of the existing work, the actuators are assumed to be perfectly embedded or bonded to the primary structure. However, it has been shown by Seeley and Chattopadhyay [12] that the control authority of smart structures can be significantly mispredicted in the presence of debonding. Recently, Chattopadhyay et al. developed an efficient and accurate framework for the analysis of adaptive composites in the presence of through-the-width delaminations [13,14]. The refined higher order theory was used to describe the displacement field. The theory accurately captures the transverse shear deformation through the thickness, which is important in anisotropic composites, particularly in the presence of discrete actuators and sensors and delaminations.

In this paper, the effects of delamination in the dynamic response of smart composite plates are investigated. A recently developed RMS based technique [14] is used to study the dynamic characteristics of both delaminated and nondelaminated composite plates with actuators. A control system is also designed to improve the dynamic response of the delaminated composite plate. The higher order refined theory developed by Seeley and Chattopadhyay is extended to model a finite rectangular delamination. The root mean square (RMS) values of the plate response to a disturbance is calculated and is used for identifying delamination location and size. An active control system is designed in an effort to make the delaminated plate behave like the healthy (nondelaminated) plate. A pole placement technique with output feedback is used in the control system design. The eigenvalues of the elastic modes are assigned to the desired values using this technique. For a given configuration, that is, with specified locations of the actuators, the effect of delamination on each piezoelectric actuator is investigated.

MATHEMATICAL FORMULATION

Finite Element Modeling of Delaminated Composite Plate

The general higher order displacement field will be extended to model composite plates with surface bonded piezoelectric materials (Fig. 1). The inplane displacements will be assumed to be effectively expressed by a cubic function through the thickness (z) and the transverse displacement will be assumed to be independent of z. To model delamination in such structures, it is necessary to partition the laminate into several different regions as shown in Fig. 1. These regions include the undelaminated region, the region above the delamination, and the region below the delamination. The interface between the undelaminated region and the delaminated region, indicated by the dashed line in Fig. 1, is denoted S. The general form of the higher order displacement field will be independently applied to each of these regions to describe displacements which account for slipping and separation due to the delamination. This general form does not satisfy the stress free boundary conditions. Therefore, it is necessary to impose the traction-free boundary condition at the top and bottom surfaces of the laminate (z = ±h/2) as well as at the delamination interface in the delaminated region. In the case of single delamination, the refined displacement field can be expressed as follows.

$$
\begin{aligned}
&\mathrm{U}_{\mathrm{i}}(x,y,z)=u_{0i}(x,y)+(z-z_{ci})[-w_{oi,x}+\alpha_i(x,y)]-4(z-z_{ci})^3\alpha_i(x,y)/(3h_i^2)\\
&\mathrm{V}_{\mathrm{i}}(x,y,z)=v_{0i}(x,y)+(z-z_{ci})[-w_{oi,y}+\beta_i(x,y)]-4(z-z_{ci})^3\beta_i(x,y)/(3h_i^2)(1)\\
&W_i(x,y,z)=w_{0i}(x,y)
\end{aligned}
$$

where U_i, V_i and W_i are the displacement functions, u_{0i}, v_{0i} and w_{0i} denote the midplane displacements of a point (x,y), $w_{0i,x}$ and $w_{0i,y}$ represent the rotation of normals to the midplanes and α_i and β_i are the additional rotations due to shear deformation. The index i stands for each region, i=1 the undelaminated region, i=2 the region above delamination and i=3 the region below delamination. The quantity z_{ci} denote the positions of the midplane for each region with respect to laminate midplane (Fig.1). In addition, continuity of displacement and traction conditions are imposed at the midplane points of each region on delamination lateral boundaries S. These continuity conditions can be exactly satisfied with the classical theory since it assumes a linear displacement distribution through the thickness. However, the displacement distribution using the refined theory is nonlinear and therefore the continuity conditions are satisfied in an average sense [12].

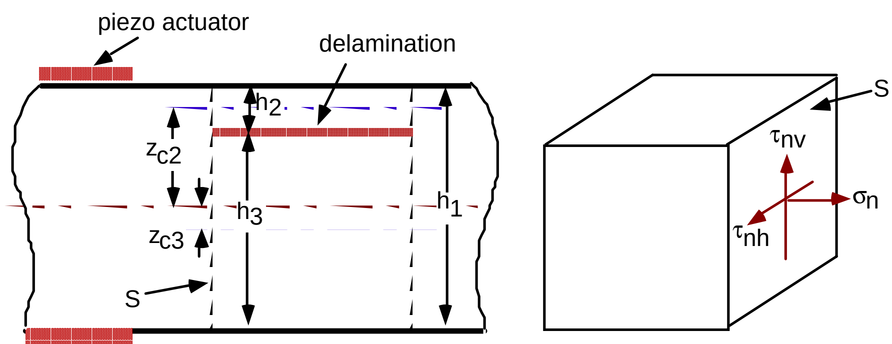


Fig. 1: Laminate cross section with delamination

For an orthotropic composite laminate with piezoelectric layers, the constitutive relationships are simplified as follows.

$$\{\sigma\}=[\overline{\mathrm{Q}}]\{\varepsilon-\Lambda\} \tag{2}$$

where Λ are the induced strains ($\Lambda_1 = \Lambda_2 = d_{31}E_3$).

The finite element method (FEM) is used to implement the refined higher order theory since it allows for the analysis of practical geometry and boundary conditions. The linear finite element equations of motion are expressed as follows.

$$\mathbf{M}\ddot{\mathbf{w}}+\mathbf{K}\mathbf{w}=\mathbf{F}+\mathbf{F}_{\mathrm{p}} \tag{3}$$

where the quantities M, K, and w denote the mass and stiffness matrices and the nodal displacement vector, respectively. The quantity F_p is the force vector due to piezoelectric actuation. Once the finite element model has been constructed, the continuity conditions are enforced using a penalty approach. More details about this procedure can be found in Ref. [12]. Bilinear shape functions are used for the inplane displacements and rotations (u, v, ϕ_X, ϕ_Y) while a 12 term cubic polynomial is used for the transverse displacement (w). The resulting four noded rectangular elements are nonconforming for computational efficiency and contain 28 degrees of freedom each.

DAMAGE DETECTION TECHNIQUE AND VIBRATION CONTROL

Root Mean Square (RMS) Values of the Response

The effects of delamination length and location on the dynamic characteristics of smart composite plates, both delaminated and nondelaminated, are investigated. The plate is subject to impulse and RMS values of the response are calculated in an attempt to understand the mechanics of damage detection. The equations of motion can be expressed as the following state equations.

$$\begin{aligned} \{\dot{x}\} &= [A]\{x\} + [B]\{u\} + \{B_w\}w \\ \{y\} &= [C]\{x\} \end{aligned} \tag{4}$$

where [A], [B] and [C] are the state matrix, the control matrix and the output matrix respectively. The quantity w is white noise and $\{B_w\}$ is noise vector. After vibration analysis, a modal reduction is performed using the first ten elastic modes. Therefore, the state matrix is of size (20×20). The RMS values of response for the elastic modes are obtained from the following equation.

$$\sigma_i^2 = [[C][X][C]^T]_{ii}, i = 1,2,.., Ns \tag{5}$$

where [C] is the output matrix, [X] is the state covariance matrix and Ns represents the number of structural states. The state covariance matrix [X] of the system is the solution of a Lyapunov equation of the form

$$[A][X] + [X][A]^T + \{B_w\}Q_w\{B_w\}^T = 0 \tag{6}$$

where Q_w is the intensity of white noise. The RMS values of the elastic modes can be converted into displacement and velocity, at the nodal points, by multiplying the covariance matrix by the matrix Φ_2 as follows

$$\sigma_{q\dot{q}i} = [\Phi_2 \, X \, \Phi_2^T]_{ii}, \quad \Phi_2 = \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \end{bmatrix} \tag{7}$$

where $\sigma_{q\dot{q}i}$ represents the RMS values of nodal displacement and nodal velocity and Φ is the eigenmatrix. The RMS values of the nodal displacements due to the disturbance are calculated for both the undelaminated and the delaminated plate models.

Active Control System Design to Minimize the Effect of Damage

The damage detection algorithm is extended to design an active control system. The goal is to reduce the RMS values and thereby minimize the effect of such damage. It is assumed that all states are available for feedback. The gain matrix of the completely controllable and observable linear dynamic system is expressed as follows

$$\{u\} = -[K_G]\{x\} \tag{8}$$

The right and left eigenvalue problems of the closed loop system can be written as

$$([A]-[B][K_G])\{\phi^c\}_i = \lambda_i^c\{\phi^c\}_i, ([A]-[B][K_G])^T\{\psi^c\}_i = \lambda_i^c\{\psi^c\}_i \tag{9}$$

where $\{\phi^c\}_i$ and $\{\psi^c\}_i$ are the right and left eigenvectors of the closed loop system, respectively, corresponding to the eigenvalue λ_i^c.

The pole placement algorithm, based on Sylvester equation, utilizes the parameter vector defined by [15]

$$\{h\}_i = [K_G]\{\phi^c\}_i \tag{10}$$

Substitution of Eq. (10) into Eq. (9) yields the following Sylvester equation

$$([A]-\lambda_i^c[I])\{\phi^c\}_i = [B]\{h\}_i \tag{11}$$

or in matrix form

$$[A][\Phi_c]-[\Phi_c][\Lambda_D] = [B][H] \tag{12}$$

where

$$\begin{aligned} [\Phi_c] &= [\{\phi^c\}_1, \{\phi^c\}_2, \dots, \{\phi^c\}_n] \\ [\Lambda_D] &= Diag(\lambda_1^c, \lambda_2^c, \dots, \lambda_n^c) \\ [H] &= [\{h\}_1, \{h\}_2, \dots, \{h\}_n] \end{aligned} \tag{13}$$

and $[\Lambda_D]$ contains the desired closed loop eigenvalues. From Eq. (11), it can be seen that

$$\{\phi^c\}_i = ([A]-\lambda_i^c[I])^{-1}[B]\{h\}_i \tag{14}$$

If λ_i^c are distinct from their open loop positions, the columns of [H] generate the corresponding closed loop eigenvectors. For the given parameter matrix [H], the closed loop modal matrix $[\Phi_c]$ can be determined by solving Sylvester equation [15]. The gain matrix can be obtained from Eq. (10) as follows

$$[K_G] = [H][\Phi_c]^{-1} \tag{15}$$

NUMERICAL EXAMPLE AND RESULTS

Results are presented for Graphite/Epoxy composite plates with five pairs of surface bonded piezoelectric actuators. The plates are clamped along one edge and other edges are free. The dynamic responses are investigated for plates with and without a small delamination (Fig. 2). The stacking sequence is $[0°/0°/90°]_S$ and each ply has a uniform thickness of *0.0000134m*.

The plate length is $0.305\ m$, width is $0.076\ m$ and total thickness is $0.000804m$. A delamination is placed at midplane in the region between $0.0152<x<0.0456\ m$, $0.0305<y<0.061\ m$. The material properties are: $E_1 = 98.0\ GPa$, $E_2 = 7.9\ GPa$, $\nu_{12} = 0.28$, $G_{12} = G_{13} = 5.6\ GPa$, $G_{23} = 2.4\ GPa$, $\rho = 1520\ Kg/m^3$ for the composite plate and $E = 63\ GPa$, $\nu = 0.31$, $G = 24.2\ GPa$, $\rho = 5000\ Kg/m^3$, $d_{12} = 250\times10^{-12}\ m/V$ for the piezoelectric material. Table 1 shows the effect of delamination on the first six natural frequencies of the composite plate. Some changes are observed in the values of the first three frequencies as a result of the small reduction in the structural stiffness due to delamination.

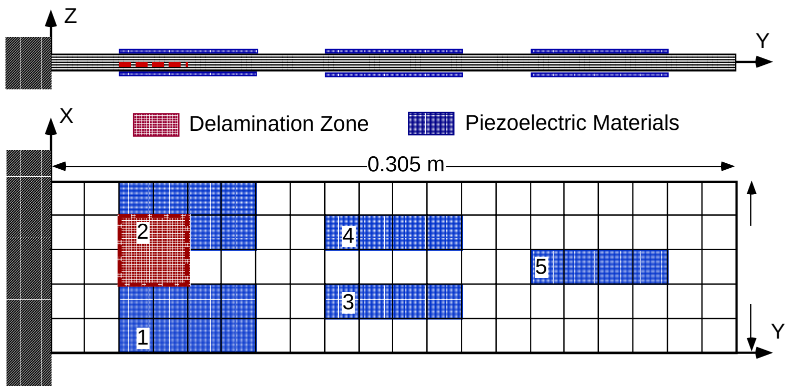


Fig. 2: Delaminated composite plate with piezoelectric actuators.

The RMS values of the nodal displacements due to the disturbance are calculated for both nondelaminated and delaminated plates. Figures 3 and 4 show the responses of the inplane displacement u, due to disturbance, for the nondelaminated and the delaminated plates, respectively. As seen from these figures, very significant differences are observed in the RMS values of inplane displacement between cases without and with delamination. In the presence of delamination, the RMS values of the response due to disturbance increases, as shown in Fig. 4, compared to the nondelaminated plate (Fig. 3). This is once again due to the reduction in plate stiffness caused by the presence of delamination. Significant jumps in the RMS values in the delamination boundaries are also observed. This information can be used to identify delmaination location and size. The RMS values of the out of plane displacement (w) were also calculated. However, it was found that the changes in RMS distributions of this displacement, between the delaminated and the nondelaminated plates, were not significant enough. Therefore, this information cannot be used to accurately capture the delamination zone. Similar observations were also made in a previous study by Chattopadhyay et al. [14].

	Nondel. Plate (Hz)	Del. Plate (Hz)
1	11.7	11.2
2	63.0	62.4
3	90.1	86.7
4	174	173
5	196	190
6	331	326

Table 1: Natural frequencies of the nondelaminated and delaminated plates.

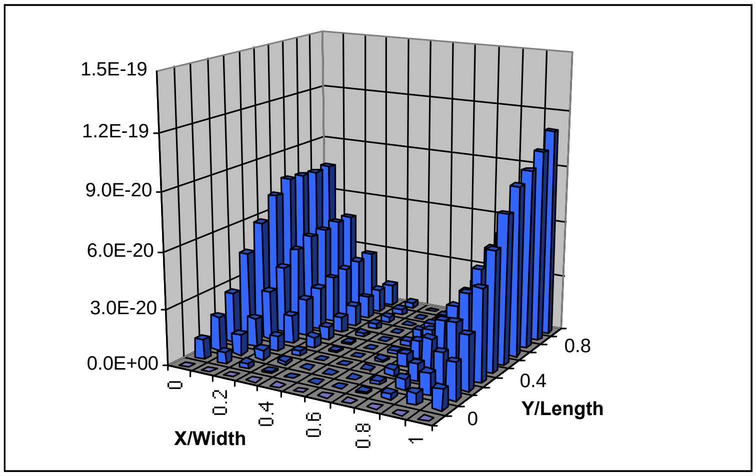


Fig. 3:RMS values of inplane displacement ,u, of the nondelaminated plate.

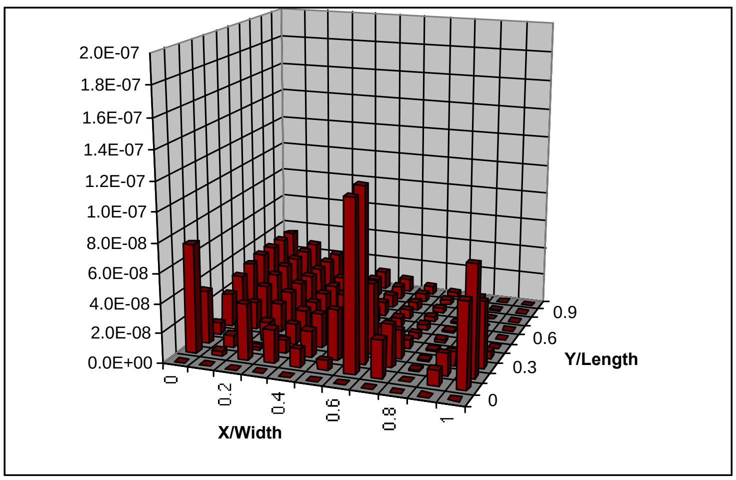


Fig. 4:RMS values of inplane displacement, u, of the delaminated plate.

Nondelaminated plate (open loop)	Delaminated plate (open loop)	Delaminated plate (closed loop)
-.1475±73.77I	-.1410±70.51I	-7.377±73.77I
-.7921±396.7I	-.7843±392.1I	-39.60±396.0I
-1.131±565.9I	-1.089±544.8I	-56.59±565.9I
-2.190±1095.I	-2.176±1088.I	-109.5±1095.I
-2.463±1231.I	-2.390±1195.I	-123.1±1231.I
-4.162±2081.I	-4.106±2053.I	-208.1±2081.I
-4.986±2493.I	-4.906±2453.I	-249.3±2493.I
-5.673±2836.I	-5.606±2803.I	-283.6±2836.I
-6.357±3178.I	-6.094±3047.I	-317.8±3178.I
-6.395±3197.I	-6.380±3190.I	-319.7±3197.I

Table 2: Changes in eigenvalues of the open loop and closed loop systems.

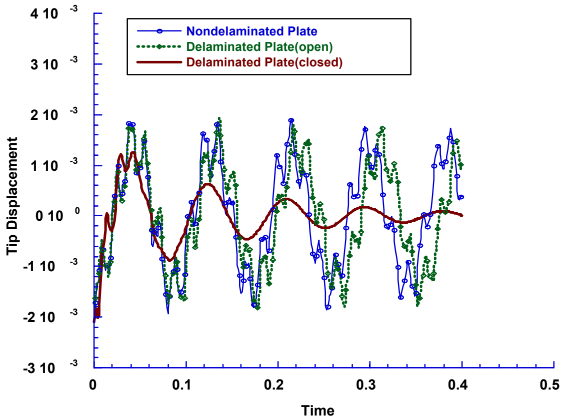
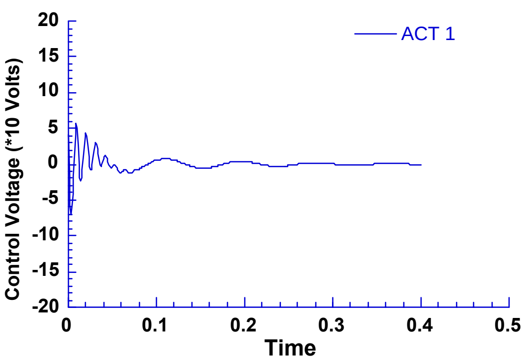
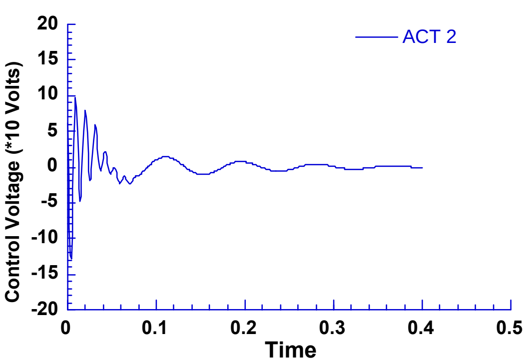


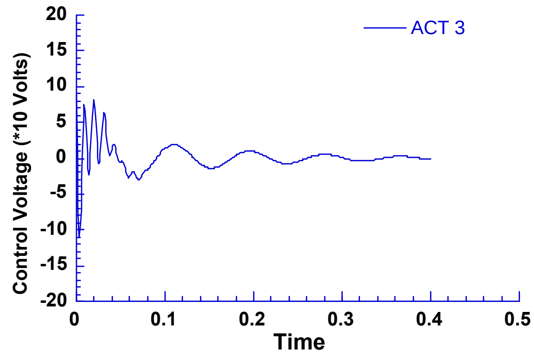
Fig. 5: Tip displacement of the composite plate due to disturbance.



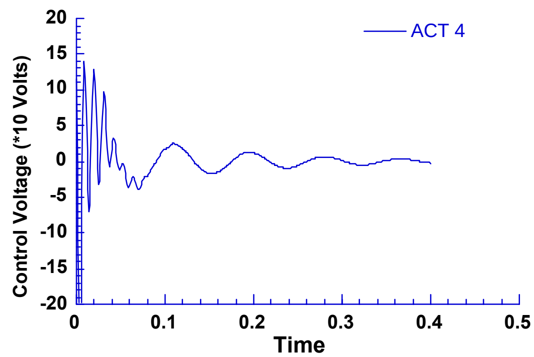
6(a)



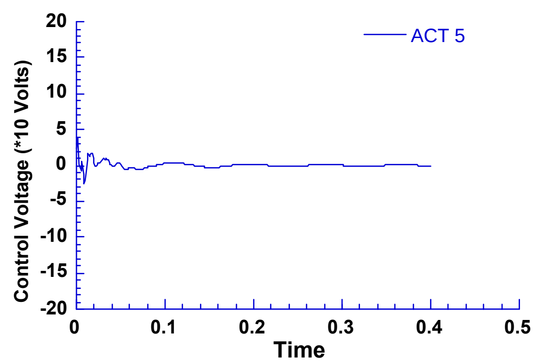
6(b)



6(c)



6(d)



6(e)

Fig. 6: Applied control voltage history.

Table 2 presents the changes in eigenvalues for both delaminated and nondelaminated plates with the application of the active control system. As seen from this Table, significant increases are observed in the real part of the closed loop eigenvalues for the delaminated plate. This indicates improved damping with closed loop control. The imaginary parts of the closed loop eigenvalues for the delaminated plate are almost identical to the corresponding open loop values for the nondelaminated plate. This implies that the natural frequencies of the two plates are almost identical. The RMS values of the first three elastic modes corresponding to the open loop system of the nondelaminated and delaminated plates are (15.227, 0.0292, 0.2093) and (21.035, 0.0142, 0.2578), respectively. With the application of the active control system, these values, for the delaminated plate, change to (0.681, 0.091, 0.0051). This indicates a very significant reduction (up to 78 percent) in the magnitude of the RMS values for the first and third modes. The RMS value of the second mode increases compared to the open loop case. However, the contribution of this mode is not significant enough to cause stability problem or degradation in the performance of the active control system. Therefore, the controller is successful in improving the dynamic characteristics of the delaminated plate.

To investigate the control performance of the designed system, the time histories of the tip displacement and the control voltages applied to the actuators are calculated. Figure 5 shows the time histories of the open loop and closed loop systems, due to disturbances, for the nondelaminated and the delaminated plates. The control system significantly improves the response of the delaminated plate compared to the nondelaminated plate (open loop). Figure 6 presents the control voltage applied to the actuators due to the disturbance. It can be seen that maximum applied voltage is less than 200 Volts implying no saturation effects.

CONCLUDING REMARKS

The dynamic characteristics of a smart composite plate in the presence of delamination are investigated. A refined higher order theory is extended to model the composite plate, with surface bonded piezoelectric actuators and sensors, in the presence of a finite delamination. The developed theory accurately captures the transverse shear deformation through the thickness, which is important in anisotropic composites, particularly in the presence of discrete actuators and sensors and delaminations. Delamination detection is investigated using Root Mean Square (RMS) values of the response of the composite plate subject to impulse. An active control system is designed to minimize the effect of delamination. The pole placement technique is used. The following observations are made from this study.

(1) Changes in the first three frequencies are observed as a result of the small reduction in the structural stiffness due to delamination.

(2) The RMS values of inplane displacement due to impulse indicate the location and size of delamination.

(3) The active control system significantly increases the damping of the delaminated plate. It also changes the natural frequencies of the delaminated plate to be almost identical to the nondelaminated plate values. Large reductions are also obtained in the RMS responses. Thus the controller is capable of improving the dynamic characteristics of the delaminated plate.

(4) Significant improvement is observed in the closed loop dynamic response of the delaminated plate compared to the open loop response of the nondelaminated plate without any saturation effect.

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