Characterization of delamination effect on composite laminates using a new generalized layerwise approach

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Abstract

A dynamic analysis method has been developed to investigate and characterize the effect due to presence of discrete single and multiple embedded delaminations on the dynamic response of composite laminated structures with balanced/unbalanced and arbitrary stacking sequences in terms of number, placement, mode shapes and natural frequencies. A new generalized layerwise finite element model is developed to model the presence of multiple finite delamination in laminated composites. The new theory accurately predicts the interlaminar shear stresses while maintaining computational efficiency.

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1. Introduction

Interlayer debonding or delamination is a prevalent form of damage phenomenon in laminated composites. Delamination can be often pre-existing or generated during service life. For example, delamination often occur at stress free edges due to the mismatch of properties at ply interfaces and it can also be generated by external forces such as out of plane loading or impact during the service life. The existence of delamination not only alters the load carrying capacity of the structure, it can also affect its dynamic response. Thus detection and quantification of delamination is an important technology that must be addressed for the successful implementation and improved reliability of such structures.

All types of damages in composite structures result in change in stiffness, strength and fatigue properties. Measurement of strength or fatigue properties during damage development is not feasible because destructive testing is required. However, stiffness reduction due to damage can be measured since damage directly affects structural response, which provides a promising method for identifying the occurrence, location and extent of the damage from measured structural dynamic characteristics. Existence of delaminations causes reduction in natural frequencies and increase in vibratory damping. Although experimental investigations are often used to study these effects, damage simulation using an accurate and efficient modeling technique can be helpful in reducing the number of expensive experiments.

Modeling and detection of delamination in composite structures has primarily been based on classical laminate theory (CLT) and first-order shear deformation theory (FSDT). This means that transverse shears are completely ignored (CLT) or are modeled using shear correction factor (FSDT). Shen and Grady [1] investigated the dynamic characteristics of a delaminated composite beam using Timoshenko beam theory. Lee [2] analyzed the free vibration characteristics of a delaminated composite beam using a layerwise theory. A refined higher order theory (HOT), developed by
Chattopadhyay and Gu [3,4], was shown to be both accurate and efficient for modeling delamination in composite plates and shells of moderately thick construction. The theory was further extended to study the effect of delamination on the dynamic response of smart composite structures [5,6]. Thornburgh and Chattopadhyay [7,8] developed a unified approach to model the presence of two types of damage, delaminations and transverse matrix cracks in composites using HOT. However, although the higher order based theories provide good accuracy at global level, they fail to satisfy stress continuity at ply interfaces. To address this issue, Barbero and Reddy [9] introduced a layerwise approach. However, the computational effort associated with such analysis increases with the increase in number of plies in the laminate. Cho and Kim [10,11] developed a higher order zigzag theory for laminated composite plates with multiple delaminations. Elasticity based models were developed by Gu and Chattopadhyay to model through-the-width delamination in simply supported composite plates [12]. Makeev and Armanios [13] analyzed laminated composites with and without delamination using a 3-D elasticity model. However, the elasticity based models although are useful in benchmark studies, they cannot be extended to the analysis of laminates with arbitrary geometry and multiple delamination. Krueger [14] developed a three-dimensional shell modeling technique for delaminations in composite laminates using the commercial software ABAQUS with four-noded quadrilateral elements. A layer reduction technique for understanding the interlaminar shear stress for composite laminates was developed by Lee and Chen [15]. Recently, Kim et al. [16] developed an improved layerwise theory to investigate interlaminar stresses in smart composite structures. The theory was shown to be both accurate and efficient in the analysis of composite shells [17] and piezoelectric composite shells [16]. Recently, this theory was extended to study the dynamic response of cross-ply laminated composite plates with multiple embedded, through-the-width delaminations [18]. In the present paper, this new layerwise model is further extended for modeling composite laminated plates with arbitrary/generalized stacking sequence and embedded multiple finite/discrete delaminations, while maintaining global and local ply level accuracy. This theory is then used to investigate the effect of delamination size, placement and number on the dynamic response of laminated composite plates. The effect of plate boundary conditions and laminate stacking sequence (including balanced and unbalanced) are also studied.

2. Mathematical theory

Shear deformation plays an important role in the response analysis of composite structures due to the material discontinuities at each interface of the laminate. To address this issue with ply level accuracy, a layer wise displacement field is used. The form of the displacement field of the perfectly bonded layers is determined by the requirements that the transverse shear stresses should vanish on top and bottom surfaces of the laminate, and should be continuous through-the-thickness. These conditions can be satisfied by layerwise functions to accommodate the complexity of zigzag-like in-plane deformation through-the-laminate thickness and to satisfy the interlaminar shear traction continuity requirement. Since transverse normal strain is negligible in bending dominant deformation of a plate, transverse deflection can be assumed to be a function of the inplane coordinates only. The first-order shear deformation is adequate for modeling the overall response of the entire laminate. To model delamination, the assumed displacement field is supplemented with Heaviside unit step functions, which allows discontinuity in the displacement field. In the following mathematical model, composite laminates with generalized stacking sequence and multiple discrete/finite delaminations are considered.

Consider a N-layered laminated composite plate with multiple delaminations as shown in Fig. 1. The displacements of a point with the coordinates (x, y, z) are described using the superposition of first-order shear deformation and layerwise functions. Following is the displacement field for a laminated plate with multiple finite delamination:

\[
U^k_i(x, y, z, t) = u_i(x, y, z, t) + \phi_i(x, y, z) + \theta^k_i(x, y, z)h(z) + \sum_{j=1}^{N-k} u^j_i(x, y, t)H(z - z_j) + \sum_{j=1}^{N-k} w^j_i(x, y, t)H(z - z_j)
\]

(1)

where the superscript \( k \) denotes the \( k \)th layer of the laminate. The subscript \( i \) denotes the coordinate \( x \) or \( y \). The unknowns are \( u_i, \phi_i, w, \theta^k_i, \psi^k_i, \dot{u}_i \) and \( \ddot{w} \). Note that \( u_i \) and \( w \) denote the displacement of the reference plane, \( \phi_i \) are rotations of the normal to the reference plane about the \( x \) and \( y \) axes, \( \theta^k_i \) and \( \psi^k_i \) are layerwise structural unknowns defined at each laminae. The terms \( \dot{u}_i \) and \( \ddot{w} \) represent possible jumps in the slipping and opening displacement and \( z_j \) denotes the delaminated interface.

Fig. 1. Schematic view of a cantilever plate with delamination.
The function $H(z-z_i)$ is Heaviside unit step function. It must be noted that in this formulation, all interfaces between layers are initially assumed to be delaminated and perfectly bonded interfaces can be easily simulated by setting $\mathbf{u}_i'$ and $w'$ to be zero in Eq. (1). Therefore, the number of delaminated layer interfaces is initially equal to the total number of plies in the laminate. The through-laminate-thickness functions, $g(z)$ and $h(z)$ are used to address the characteristics of in-plane zigzag coordinates of the bottom and the top surfaces, respectively. The transverse shear stress continuity conditions are imposed at the delaminated interface as follows:

$$g(z) = \sinh(z/h)$$
$$h(z) = \cosh(z/h)$$

where the functions $g(z)$ and $h(z)$ render higher order odd and even distributions, respectively.

The above displacement fields lead to a total of $5 + 4N + 3(N - 1)$ structural unknowns where $N$ is number of layers. The total number of structural unknowns is dependent on the number of layers and delaminations, implying that computational effort will increase greatly if multilayered laminates are used. To reduce the number of variables, the conditions of zero surface traction at the top and bottom surfaces, and continuity of transverse shear stresses and in-plane displacements at interlaminar surfaces are imposed. The surface traction free boundary conditions on the outer free surfaces are written as follows:

$$t_{x_i}^z(x,y,z) = 0$$
$$t_{y_i}^z(x,y,z_{N+1}) = 0$$

where the quantities $z_i$ and $z_{N+1}$ denote the thickness coordinates of the bottom and the top surfaces, respectively. For orthotropic layer, the transverse shear stresses depend on transverse shear strains. Thus, the traction free condition can be rewritten as follows:

$$\phi_1 + g_z \psi_1^z + h_z \theta_1^z + w^z_j = 0 \quad \text{at} \quad z = z_i$$
$$\phi_1 + g_z \psi_1^z + h_z \theta_1^z + w^z_j + \sum_{j=1}^{N-1} w^z_j H(z-z_j) = 0$$

at $z = z_{N+1}$

where $\theta$ and $\phi$ denote partial derivatives with respect to the $x$, $y$ and $z$-coordinates.

At the perfectly bonded interfaces, transverse shear stresses are continuous. At the delaminated interface, transverse shear stresses are zero. In the present theory, transverse shear stress continuity conditions are assumed to be satisfied at the delaminated interface because zero shear stresses also satisfy continuity of stresses. Therefore, transverse shear stress continuity conditions are imposed at the delaminated interface as follows:

$$t_{x_i}^z(x,\beta,z_{k+1}) = t_{x_i}^{k+1}(x,\beta,z_{k+1})$$

These continuities can be further expressed using kinematic relations and Hooke’s law as follows:

$$Q_{1}^{k} \left[ \phi_1 + g_z(z_{k+1}) \theta_1^k + h_z(z_{k+1}) \psi_1^k + w^z_x + \sum_{j=1}^{k} w^z_j H(z-z_j) \right]$$

$$+ Q_{45}^{k} \left[ \phi_2 + g_z(z_{k+1}) \theta_2^k + h_z(z_{k+1}) \psi_2^k + w^z_y + \sum_{j=1}^{k} w^z_j H(z-z_j) \right]$$

$$= Q_{55}^{(k+1)} \left[ \phi_1 + g_z(z_{k+1}) \theta_1^{k+1} + h_z(z_{k+1}) \psi_1^{k+1} + w^z_x + \sum_{j=1}^{k+1} w^z_j H(z-z_j) \right]$$

$$+ Q_{45}^{(k+1)} \left[ \phi_2 + g_z(z_{k+1}) \theta_2^{k+1} + h_z(z_{k+1}) \psi_2^{k+1} + w^z_y + \sum_{j=1}^{k+1} w^z_j H(z-z_j) \right]$$

$$= Q_{1}^{k} \left[ \phi_1 + g_z(z_{k+1}) \theta_1^k + h_z(z_{k+1}) \psi_1^k + w^z_x + \sum_{j=1}^{k} w^z_j H(z-z_j) \right]$$

$$+ Q_{45}^{k} \left[ \phi_2 + g_z(z_{k+1}) \theta_2^k + h_z(z_{k+1}) \psi_2^k + w^z_y + \sum_{j=1}^{k} w^z_j H(z-z_j) \right]$$

$$= Q_{55}^{(k+1)} \left[ \phi_1 + g_z(z_{k+1}) \theta_1^{k+1} + h_z(z_{k+1}) \psi_1^{k+1} + w^z_x + \sum_{j=1}^{k+1} w^z_j H(z-z_j) \right]$$

$$+ Q_{45}^{(k+1)} \left[ \phi_2 + g_z(z_{k+1}) \theta_2^{k+1} + h_z(z_{k+1}) \psi_2^{k+1} + w^z_y + \sum_{j=1}^{k+1} w^z_j H(z-z_j) \right]$$

where $Q_{45}^{(k)}$, $Q_{55}^{(k)}$ and $Q_{55}^{(k+1)}$ are reduced stiffness of the $k$th lamina. Furthermore, at the perfectly bonded interfaces, the in-plane displacements are continuous at each interface. The displacements are also assumed to be continuous at the delaminated interface, allowing slipping effect by $\mathbf{u}$. The continuity conditions of in-plane displacements at the $k$th interface can be expressed as follows:

$$g(z_{k+1}) \theta_1^k + h(z_{k+1}) \psi_1^k = g(z_{k+1}) \theta_1^{k+1} + h(z_{k+1}) \psi_1^{k+1}$$

Using Eqs. (4), (6) and (7), the structural unknowns of the $k$th layer are related to those of the $(k-1)$th
layer. Thus, for an N-layer laminated composite with multiple delaminations, 4N constraint conditions are obtained. By substituting the 4N equations into the assumed displacement field (Eq. (1)), the in-plane displacements of the delaminated composite laminate are expressed as follows:

\[ U^i_k(x, y, z, t) = u_1^i + A^i_1(z)\phi_1 + B^i_1(z)\phi_2 + C^i_1(z)w_x + D^i_1(z)w_y + \sum_{j=1}^{N-1}\bar{u}^i_jH(z-z_j) \]

\[ U^j_k(x, y, z, t) = u_2 + A^j_2(z)\phi_1 + B^j_2(z)\phi_2 + C^j_2(z)w_x + D^j_2(z)w_y + \sum_{j=1}^{N-1}\bar{u}^j_jH(z-z_j) \]

where

\[ A^i_1(z) = z + a^i_1g(z) + \epsilon^i_1h(z) \quad A^j_1(z) = a^j_1g(z) + \epsilon^j_1h(z) \]

\[ B^i_1(z) = b^i_1g(z) + f^i_1h(z) \quad B^j_1(z) = z + b^j_2g(z) + f^j_2h(z) \]

\[ C^i_1(z) = a^i_2g(z) + \epsilon^i_1h(z) \quad C^j_2(z) = a^j_2g(z) + \epsilon^j_1h(z) \]

\[ D^i_1(z) = b^i_2g(z) + z + b^i_2g(z) + f^i_2h(z) \]

The layerwise coefficients are obtained from the 4N constraint equations and are expressed in terms of laminate geometry and material properties. The coefficients \( a^i_k, b^i_k, \epsilon^i_k \) and \( f^i_k \) are scalar quantities at each layer \( i = 1, 2 \). However, coefficients \( c^i_k, d^i_k, g^i_k \) are 1 \( \times \) \( D \) row vectors at each layer and describe the slipping and opening effect due to delamination, \( D \) being the number of embedded delamination. The displacement field, ranging from the first layer to the \( N \)th layer, can now be expressed in terms of the variables \( u_1, u_2, w, \phi_1, \phi_2, \bar{u}_1, \bar{u}_2 \) and \( \bar{w} \). Thus the total number of unknowns is dependent on the number of delamination but independent of the number of layers in the laminate, resulting in significant computational saving.

### 3. Finite element implementation

The theory is implemented using finite element method to address arbitrary boundary conditions, stacking sequence and delamination location. In this work, four-noded plate element is used with linear Lagrange interpolation functions to model the in-plane unknowns and Hermite cubic interpolation function is used for the out-of-plane unknowns. The primary displacement unknowns are expressed in terms of nodal values and shape functions as follows:

\[ (u_1, u_2, \phi_1, \phi_2, \bar{u}_1, \bar{u}_2) = \sum_{m=1}^{n}N_m[(u_1)_m, (u_2)_m, (\phi_1)_m, (\phi_2)_m, (\bar{u}_1)_m, (\bar{u}_2)_m] \]

\[ w = \sum_{m=1}^{n}(H_m(w)_m + H_{mm}(w_x)_m + H_{mm}(w_y)_m) \]

\[ \bar{w} = \sum_{m=1}^{n}(H_m(\bar{w})_m + H_{mm}(\bar{w}_x)_m + H_{mm}(\bar{w}_y)_m) \]

where \( n \) is the number of nodes in an element, \( N_m \) is the Lagrange interpolation function and \( H_m, H_{mm} \) are Hermite interpolation functions. The relationship between displacement unknowns and nodal unknowns in Eq. (10) can be expressed by the following matrix form:

\[ \{u\} = [N]\{d\} \]

The nodal unknowns and interpolation matrix are defined in Appendix A. The additional nodal unknowns for delamination, \( \bar{u}_1, \bar{u}_2, \bar{w}, \bar{w}_x, \bar{w}_y \), and \( \bar{w}_x, \bar{w}_y \), are going to be zero in the healthy plate. These unknowns are fixed or free at the boundaries of discrete delamination.

Based on the previously described field assumptions and kinematic relations, the element displacement field \( u(x, y, z, t) \) and the strain field \( \varepsilon(x, y, z, t) \) can be written as follows:

\[ u(x, y, z, t) = L_u\varepsilon(x, y, t) \]

\[ \varepsilon(x, y, z, t) = L_e\varepsilon(x, y, t) \]

The higher order operators, \( L_u \) and \( L_e \), are defined in Appendix B. The total number of generalized nodal unknowns is 28 for a healthy laminated plate element and 28 + 5 \( \times \) \( D \) for a delaminated element.

The equations of motion are obtained using Hamilton’s principle as follows:

\[ \int_0^T \left\{ \int_V (\rho \ddot{u} \delta u + \sigma_{ij} \delta \varepsilon_{ij}) dV - \int_{\Gamma_e} r \delta u d\Gamma \right\} dt = 0 \]  

Substitution of Eqs. (11) and (12) into (13), results in the following finite element representation of the equations of motion for the free vibration problem:

\[ ([K] - \omega^2[M])\{d\} = 0 \]

where \([K]\) and \([M]\) are the stiffness and mass matrices, respectively. The parameter \( \omega \) and \( \{d\} \) denote the natural frequency and the associated eigenvector of nodal displacements, respectively. The stiffness and mass matrices are defined as follows:
\begin{align}
[K] &= \int_V B^T Q B \, dV \\
[M] &= \int_V B^T Q B \, dV
\end{align}

with the following definition of the operators:

\begin{align}
B_0 &= L_0 N \\
B_k &= L_e N
\end{align}

4. Results and discussion

Several numerical investigations are conducted to characterize the effect of single and multiple discrete/finite delamination in generalized lay-up laminated composite plates. The effects of number and placement of delamination on mode shapes and natural frequencies are investigated. The influence of laminate stacking sequence and plate boundary conditions on the dynamic response of delaminated plates are also studied. Comparisons are made between the response of a healthy composite plate and plates with single/multiple finite delamination. Detailed validation studies of the developed theory with those obtained using higher order theory, available and new experimental results, for a cross-ply laminate, have been documented in Ref. [18].

Effects of balanced and unbalanced stacking sequence for composite laminated plate are studied first and the natural frequency results obtained by the present layerwise theory are compared with results from HOT. A carbon-epoxy cantilever composite plate, with dimensions 12.7 cm (5 in.) length and 1.27 cm (0.5 in.) width is considered in this numerical study (Fig. 1). The ply thickness is 0.0127 cm (0.005 in.) and the material width is considered in this numerical study (Fig. 1). The dimensions 12.7 cm (5 in.) and 1.27 cm (0.5 in.) are presented for the plate without and with a single delamination. The effects due to the presence of delamination follow the same pattern as those observed in the balanced laminate. While close proximity with the HOT results is once again observed, as in the case of the balanced laminate, deviation with HOT is observable in the second bending mode for delamination present near the surface. When the natural frequencies of the balanced stacking sequence ([45/−45]3s) and unbalanced stacking sequence ([0/45]3s) composite laminated plates are compared, it is observed that the natural frequencies for the bending modes are lower in the balanced stacking sequence of ([45/−45]3s). Results are presented for the undelaminated (healthy) plate and the plate with a single delamination of size 5.08 cm (40% delamination) as shown in Fig. 1. The results are computed with the delamination located at various ply interfaces. The length to thickness ratio is assumed as \( L/h = 125 \). The delaminated element has additional nodal unknowns \( \bar{u}_{ij}, \bar{w}_{ij}, \bar{w}_{i}, \bar{w}'_{ij} \) and \( \bar{w}'_{ij} \). These additional nodal unknowns are fixed along the line a (Fig. 1), and are free along line b (Fig. 1) at the boundaries of delamination. In Fig. 2, 10, 11, 12 and 13 refer to the ply level location of the delamination, measured from the midsurface. For example, 10 refers to a delamination at the midsurface and 11 refers to a delamination at the first interface measured from the midsurface. It is also noted that the natural frequencies of the delaminated plate shift from the healthy one, more when the delamination is located near the midplane and less when the delamination is located near the surface and away from the midplane.

This trend is observed in the first three modes using the present analysis and in the first bending and twisting modes using HOT and not in the second bending mode as predicted by HOT. The discrepancy suggests that the ply level information from the present layerwise theory provide more accurate description of strain field, resulting in more accurate prediction of natural frequencies. Fig. 3 compares the natural frequencies for a cantilever plate with an unbalanced stacking sequence ([0/45]3s). The material properties and plate geometry are the same as in the previous case. Once again, results are presented for the plate without and with a single delamination; the location varying through-the-thickness direction. The effects due to the presence of delamination follow the same pattern as those observed in the balanced laminate. When close proximity with the HOT results is once again observed, as in the case of the balanced laminate, deviation with HOT is observable in the second bending mode for delamination present near the surface. When the natural frequencies of the balanced stacking sequence ([45/−45]3s) and unbalanced stacking sequence ([0/45]3s) composite laminated plates are compared, it is observed that the natural frequencies for the bending modes are lower in the balanced

![Fig. 2. Natural frequencies of a cantilever plate ([45/−45]3s) with a 40% delamination (L/h = 125).](image-url)
laminated plate whereas the opposite hold true for the twisting modes. The presence of delamination at any ply level did not alter this trend in the natural frequencies.

Comparisons are then made between a thick \((L/h = 25)\) and a thin \((L/h = 125)\) cantilever \([0/90]_2\) laminated plate with a 5.08 cm (40%) delamination. Fig. 4 presents the natural frequencies associated with the first bending mode, second bending mode and the first twisting mode for thick and thin laminated composite plates with single finite delamination. The natural frequency shift trends obtained by the present layerwise theory are found to be similar in pattern for both of the thick and thin composite laminated plates with embedded delamination. However, the patterns of natural frequency shift predicted by HOT are different for the second bending mode and the first twisting mode from those obtained by the present layerwise theory for the thin plate. The prediction of transverse shear stress is more accurate in the present method than HOT. This is reflected in the differences observed in the higher modes as predicted by HOT especially when the delamination is near the surface.

Next, numerical investigations are conducted to study the effect stacking sequence on the dynamic response of delaminated plates. A finite element mesh consisting of 30 \(\times\) 4 four-noded plate elements is used to model single and multiple delamination. The geometry of the carbon cyanate laminated composite cantilever plate with single delaminations is shown in Fig. 1. The material properties for the laminated composite plate are as following: \(E_1 = 380\) GPa, \(E_2 = 16.6\) GPa, \(G_{12} = 4.2\) GPa, \(\rho = 1800\) kg/m\(^3\), \(\nu_{12} = 0.31\), \(\nu_{23} = 0.42\).
The dimensions of the cantilever plate are 30 cm length, 5 cm width, and 0.218 cm thickness. First, a single delamination of size $6 \text{ cm} \times 5 \text{ cm}$ (20% delamination) at a distance of 12 cm from the fixed end is considered. The delamination is located at the I2 ply level. Table 1 shows the first five natural frequencies of healthy and delaminated plates with stacking sequences, $[0/90]_4s$, $[30/-30]_4s$, $[45/-45]_4s$, respectively. The order of natural frequency modes changes when the stacking angle changes. For example, the first twisting mode shifts from the second natural frequency for $[0/90]_4s$ to the fifth natural frequency for $[45/-45]_4s$. This is due to the change in the system stiffness caused by the change in ply orientations. The effects of stacking sequences on the first flexural and twisting natural frequency modes are presented in Fig. 5 for the aforementioned composite laminated plates with different stacking sequences and single delamination with respect to healthy plates. Relative differences in frequency shifts are demonstrated using the formula, $(\omega_d - \omega_h)/\omega_h \times 100$, where $\omega_d$ and $\omega_h$ represent frequencies of delaminated and healthy plate, respectively. The trends in natural frequency shifts, from healthy to delaminated plate, is also influenced by the changes in stacking sequence. As seen from this figure, the frequency shifts increase with increase in ply orientation, particularly for the twisting modes. Such observation can be useful in the development of damage indices for delamination detection using vibrational techniques.

Table 2 shows the first five natural frequencies of healthy and multiple delaminated plates for plates with different stacking sequences, $[0/90]_4s$, $[30/-30]_4s$, $[45/-45]_4s$. The two delaminations are located at I2 and I4 ply levels. The trends in natural frequencies are similar to the case of single delamination described before. Fig. 6 shows the effect of stacking sequences on the natural frequency shifts. The frequency shifts are larger compared to the single delamination case, as expected. However, the trends in natural frequency shift, with increase in stacking angle, are similar to the single delamination case.

Finally, a c–c–c–c (all four sides clamped) carbon cyanate laminated square plate with discrete delaminations of arbitrary size, number and in-plane location is investigated as shown in Fig. 7. The laminated composite plate considered is of the dimension $30 \text{ cm} \times 30 \text{ cm}$, thickness 0.24 cm, and the stacking sequence is $[0/90]_4s$. A finite element mesh consisting of $15 \times 15$ four-noded plate elements is used to model the plate with arbitrary discrete delamination. Table 3 shows the first five natural frequencies of plates with (i) no delamination, different stacking sequences and single delamination with respect to healthy plates. Relative differences in frequency shifts are demonstrated using the formula, $(\omega_d - \omega_h)/\omega_h \times 100$, where $\omega_d$ and $\omega_h$ represent frequencies of delaminated and healthy plate, respectively. The trends in natural frequency shifts, from healthy to delaminated plate, is also influenced by the changes in stacking sequence. As seen from this figure, the frequency shifts increase with increase in ply orientation, particularly for the twisting modes. Such observation can be useful in the development of damage indices for delamination detection using vibrational techniques.

Table 2

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The dimensions of the cantilever plate are 30 cm length, 5 cm width, and 0.218 cm thickness. First, a single delamination of size $6 \text{ cm} \times 5 \text{ cm}$ (20% delamination) at a distance of 12 cm from the fixed end is considered. The delamination is located at the I2 ply level. Table 1 shows the first five natural frequencies of healthy and delaminated plates with stacking sequences, $[0/90]_4s$, $[30/-30]_4s$, $[45/-45]_4s$, respectively. The order of natural frequency modes changes when the stacking angle changes. For example, the first twisting mode shifts from the second natural frequency for $[0/90]_4s$ to the fifth natural frequency for $[45/-45]_4s$. This is due to the change in the system stiffness caused by the change in ply orientations. The effects of stacking sequences on the first flexural and twisting natural frequency modes are presented in Fig. 5 for the aforementioned composite laminated plates with different stacking sequences and single delamination with respect to healthy plates.

![Fig. 5. Relative natural frequency shift showing the effect of stacking sequences in the cantilever plates with single delamination with respect to healthy plates.](image-url)
(ii) centrally located discrete delamination of size 10 cm × 10 cm (D1), (iii) corner located delamination of 10 cm × 10 cm, located 4 cm away from each edge (D2) and a plate with a two equal sized (10 cm × 10 cm) discrete delamination, symmetrically located at each corner side (D3). In all of these cases (D1, D2 and D3), the delamination is located at I2 ply level and is through-the-thickness. The boundaries on all sides of the discrete delamination are considered to be fixed. It is observed, comparing cases D2 and D3 with respect to the undelaminated plate, that the frequency shifts due to the presence of delamination is significantly more pronounced for the case with multiple delamination when compared to the case with a single delamination. However, comparing cases D1 and D3, it can be seen that the mode shapes associated with $o_{12}$ and $o_{21}$ modes do not follow the same pattern due to the location of the delamination. These modes are affected more by the centrally located delamination than the quarter sized located two delaminations. The other modes however follow the same pattern as seen before. This signifies the importance of the in-plane location of the delamination, which affect some of the modes more than others.

Fig. 8 shows the first four normalized mode shapes of the aforementioned square plate without delamination. Fig. 9 shows the contour plot of the difference in normalized mode shapes between the healthy plate and the plate with a centrally located discrete delamination (case D1). As seen from this figure, the mode shapes are definitely affected by the presence of delamination. The differences are symmetric due to the symmetry of plate geometry and delamination location, which is clearly observable in the contour plots. Fig. 10 shows the difference in normalized mode shapes between healthy plate and the plate with quarter sized discrete delamination (case D2). The location of delamination is easily observable in the contour plots corresponding to mode (1,1) and mode (1,2). It is observed that mode (2,1) and mode (2,2) shows symmetric distribution of deflection and hence the location of the quarter delamination is difficult to discern from these plots. Fig. 11 shows the difference in normalized mode shapes between healthy plate and the plate with two discrete delaminations (case D3). The effect of delamination is easily observed in the contour plots of mode (1,1), mode (1,2) and mode (2,2). It is hard to discern the location of the delaminations from mode (2,1) due to the nature of the mode shape and the locations of the two delaminations. These results provide valuable information on the effects of the

---

Table 3
Natural frequencies of discrete single and multiple delamination of arbitrary size

<table>
<thead>
<tr>
<th>$o_{mn}$</th>
<th>Undel.</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$o_{11}$</td>
<td>409.80</td>
<td>408.15</td>
<td>397.16</td>
<td>386.05</td>
</tr>
<tr>
<td>$o_{12}$</td>
<td>819.52</td>
<td>734.94</td>
<td>804.00</td>
<td>789.19</td>
</tr>
<tr>
<td>$o_{21}$</td>
<td>871.50</td>
<td>785.23</td>
<td>857.64</td>
<td>845.89</td>
</tr>
<tr>
<td>$o_{22}$</td>
<td>1133.7</td>
<td>1097.6</td>
<td>1112.9</td>
<td>1093.7</td>
</tr>
<tr>
<td>$o_{13}$</td>
<td>1516.8</td>
<td>1424.0</td>
<td>1370.0</td>
<td>1284.4</td>
</tr>
</tbody>
</table>

Fig. 7. Size and location of arbitrary delamination in square plate.
Fig. 8. Mode shapes of square composite plate.

Fig. 9. The differences of mode shapes between healthy and D1.
Fig. 10. The differences of mode shapes between healthy and D2.

Fig. 11. The differences of mode shapes between healthy and D3.
number and in-plane position of the delamination on the dynamic characteristics of the plate.

5. Concluding remarks

A method has been developed to investigate the dynamic response of composite plates with arbitrary stacking sequence and multiple discrete delaminations. The mathematical model is based on a newly developed layerwise approach that is both efficient and accurate. The procedure has been used to characterize the effect of delaminations on the frequencies and mode shapes of both thick and thin laminates with balanced, unbalanced, angle and cross-ply stacking sequences. The effects of number, placement, and size of delamination on the dynamic response were studied. The following important observations are made from the present study:

1. The natural frequencies obtained using the developed model correlates well with both experimental results and results obtained using a higher order theory.
2. For the same geometry, the bending natural frequencies are lower in the balanced laminate and the natural frequencies of the twisting modes are lower in the unbalanced case. The presence of delamination at any ply level did not alter this trend in natural frequencies.
3. The trends in natural frequency shift, from healthy to delaminated plate, are similar for both thick and thin constructions.
4. The natural frequency shifts increase with increase in ply orientation angle.
5. The location of delamination can be detected through the difference in mode shapes, more easily when multiple discrete delaminations are present.

Acknowledgements

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Appendix A

Nodal unknowns and interpolation matrix are defined as follows:

\[
\{u^u\} = \begin{bmatrix} u_1, u_2, w, \phi_1, \phi_2, \bar{u}_1, \bar{u}_2, \bar{w} \end{bmatrix}^T \\
\{d\} = \begin{bmatrix} \ldots, u_{1i}, u_{2j}, w, \phi_{1l}, \phi_{2k}, \bar{u}_{1i}, \bar{u}_{2j}, \bar{w}, \bar{w}_1, \bar{w}_2, \ldots \end{bmatrix}^T
\]

\(N\) is the total number of delaminations, \(\mathbf{0}^o\) is the 1 x \(D\) null row vector, \(\mathbf{N}_i^o, \mathbf{\bar{u}}_i^o, \mathbf{\bar{u}}_j^o\) and \(\mathbf{\bar{w}}_i^o\) are 1 x \(D\) row vector and defined as follows:

\[
\mathbf{N}_i^o = [N_i, \ldots, N_i] \\
\mathbf{\bar{u}}_i^o = [H_i, \ldots, H_i] \\
\mathbf{\bar{u}}_j^o = [H_{ij}, \ldots, H_{ij}] \\
\mathbf{\bar{w}}_i^o = [H_{ij}, \ldots, H_{ij}]
\]

Appendix B

The higher order operators, \(\mathbf{L}_s\) and \(\mathbf{L}_c\), are defined as follows:

\[
\mathbf{L}_s = \begin{bmatrix} 1 & 0 & C_1^s \xi_1 + D_1^s \xi_1 & A_1^s & B_1^s & \mathbf{\bar{u}}_1^o & \mathbf{\bar{w}}_1^o & \mathbf{\bar{w}}_1^o \xi_1 + \mathbf{\bar{w}}_1^o \xi_1 \\
0 & 1 & C_2^s \xi_2 + D_2^s \xi_2 & A_2^s & B_2^s & \mathbf{\bar{u}}_1^o & \mathbf{\bar{w}}_1^o & \mathbf{\bar{w}}_1^o \xi_2 + \mathbf{\bar{w}}_1^o \xi_2 \\
0 & 0 & 1 & 0 & 0 & \mathbf{\bar{u}}_1^o & \mathbf{\bar{w}}_1^o & \mathbf{\bar{w}}_1^o & \mathbf{\bar{w}}_1^o & \mathbf{\bar{w}}_1^o & \mathbf{\bar{w}}_1^o \xi_1 + \mathbf{\bar{w}}_1^o \xi_1 \\
\end{bmatrix}
\]

\(D\) is the total number of delaminations, \(\mathbf{0}^o\) is the 1 x \(D\) null row vector, \(\mathbf{\bar{u}}_i^o, \mathbf{\bar{u}}_j^o, \mathbf{\bar{u}}_j^o\) and \(\mathbf{\bar{w}}_i^o\) are 1 x \(D\) row vector and defined as follows:

\[
\mathbf{N}_i^o = [N_i, \ldots, N_i] \\
\mathbf{\bar{u}}_i^o = [H_i, \ldots, H_i] \\
\mathbf{\bar{u}}_j^o = [H_{ij}, \ldots, H_{ij}] \\
\mathbf{\bar{w}}_i^o = [H_{ij}, \ldots, H_{ij}]
\]
References