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A Discrete Semianalytical Procedure for Aerodynamic Sensitivity Analysis Including Grid Sensitivity

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Abstract—A discrete semianalytical sensitivity analysis procedure has been developed for calculating aerodynamic design sensitivities. The sensitivities are numerically calculated using direct differentiation of the discretized flow equations. A new approach has been developed and employed to calculate the sensitivities of the discretized grid, which are integral to the calculation of the aerodynamic sensitivities. Representative results from the grid sensitivity analysis and semianalytical sensitivity analysis procedures are compared with those obtained from the finite difference approach to establish their efficiency and accuracy. The developed procedures offer significant savings in computing time over the finite difference approach, thus allowing the use of comprehensive analysis procedures in design optimization.

Keywords—Sensitivity analysis, Semianalytical, Aerodynamic sensitivity, Grid sensitivity, Optimization.

INTRODUCTION

Recently, there has been significant interest in the use of formal optimization techniques in the design of aerospace vehicles. In order for those designs to be meaningful, it is necessary to couple advanced and often complex analysis techniques inside the closed-loop optimization procedure. An accurate solution of the flow field necessitates the use of comprehensive solution techniques. Computational Fluid Dynamics (CFD) has advanced rapidly with the development of numerous numerical algorithms, and detailed analyses of many complex flow fields associated with practical engineering systems are now possible using supercomputers. However, viscous-compressible flow simulations of aircraft configurations can require several CPU hours per steady-state solution [1]. Therefore, for advanced flow analysis procedures to be included within a multidisciplinary optimization environment, efficient sensitivity analysis techniques must be developed.

An essential ingredient in gradient-based optimization techniques is the calculation of design sensitivities, also called Design Sensitivity Analysis (DSA). Three different techniques exist for performing DSA, namely

(1) the finite difference technique,
(2) the direct differentiation method, and
(3) the adjoint variable method.

The simplest and the most popular among these three methods is the finite difference method. The use of this method is associated with several calls to the analysis routines. Although this

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Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX technique is conceptually simple, the associated computational cost is prohibitive when used in an optimization problem involving complex analysis procedures and a large number of design variables. Therefore, it is necessary to develop efficient techniques to calculate aerodynamic sensitivities so that advanced CFD codes may be utilized as practical design tools within multidisciplinary optimization environments.

The direct differentiation and the adjoint variable approaches are categorized as semianalytical sensitivity analysis techniques. They are based on simple chain rule of differentiation and offer significant savings in computational time. Although conceptually simple, efficient implementation of these two techniques requires in-depth knowledge of the analysis techniques and the solution procedure. In both of these techniques, the actual governing equations are differentiated with respect to the design variables. The direct differentiation approach yields a large system of equations involving the desired sensitivities which can be solved directly. In the adjoint variable approach, adjoint variables are obtained as the solution to an adjoint problem. The adjoint variables are then used to calculate the sensitivities. These two techniques are equivalent and yield identical results for the sensitivities. The direct differentiation is generally used in problems with a large number of response functions, and the adjoint variable approach is used in problems with a large number of design variables. If the governing equations are differentiated prior to their discretization, the semianalytical approach is called a continuous sensitivity approach. In the continuous sensitivity approach, the sensitivities are calculated using a numerical algorithm similar to the one used for obtaining the flow solution. Therefore, the continuous sensitivity approach needs to be modified, depending upon the governing equations that are differentiated. In the discrete sensitivity approach, the discretized form of the governing equations, obtained from the numerical algorithm, are differentiated. Although there is a need for solving a large system of equations, the procedure can easily be adapted to different numerical algorithms.

Due to the important nature of the subject, significant research efforts have been reported over the last few years in developing semianalytical techniques for DSA. Although more significant efforts have been reported in structural DSA [2,3], recently, advances have been made in developing semianalytical techniques for aerodynamic DSA [4–10]. In all of these works, either the direct differentiation technique or the adjoint variable technique was used. Carlson and Elbanna [4] have used the direct differentiation technique to differentiate the discretized transonic small perturbation equations to obtain aerodynamic sensitivities. Jameson *et al.* [5] have proposed a continuous sensitivity approach using the adjoint variable method to calculate aerodynamic sensitivities. Baysal *et al.* [6,7] have performed discrete sensitivity analysis using the Euler equations. Taylor *et al.* [8] and Newman *et al.* [9] have developed a semianalytical sensitivity analysis procedure for the thin layer Navier-Stokes equations using an incremental strategy. Chattopadhyay and Pagaldipti [10] have used the direct differentiation approach to perform aerodynamic sensitivities in a multilevel optimization procedure using the parabolized Navier- Stokes equations.

Two main ingredients in an aerodynamic sensitivity analysis procedure are:

(1) the calculation of the sensitivities of the discretized flow variables and
(2) the calculation of the sensitivities of the computational grid with respect to the aerodynamic design variables.

It has been well recognized that the sensitivities of the flow variables are dependent upon the sensitivities of the computational grid [5–10]. However, in most of the aforementioned work, brute force finite difference techniques were used to calculate the grid sensitivities. Very few formal investigations have been reported on the development of analytical or semianalytical techniques for computing grid sensitivities. High quality elliptic and hyperbolic grid generation codes are often used for generating meshes for aircraft configurations [11]. The use of the finite difference method for calculating grid sensitivities can be computationally prohibitive in such situations. Korivi *et al.* [12] developed a grid sensitivity analysis wherein the Jacobian matrix of the entire grid with respect to the grid points on the boundary of the domain is calculated.

The sensitivities of the surface grid points are calculated using an elastic membrane analogy to represent the computational domain, and the surface grid sensitivities are calculated from a structural analysis code using the finite elements method. Extension of this technique to complex three-dimensional flow fields can be extremely complicated and time consuming. Further, the use of an additional structural analysis code increases computing time. Sadrehaghighi *et al.* [13] proposed an analytical approach for calculating grid sensitivities in which algebraic grid generation is performed using transfinite interpolation and surface parameterization in terms of design variables. The transfinite interpolation equations are analytically differentiated to obtain the grid sensitivities. The most general parameterization of the boundaries would require the specification of every grid point on the boundary. This, however, is impractical from a computational point of view. A quasi-analytical parameterization is used in [13] which allows the aircraft component to be specified by a relatively smaller number of parameters. However, the technique does not offer a great amount of generality because most CFD codes use complex grids which are generated using methods based on partial differential equations.

In this paper, the grid sensitivity parameters are efficiently calculated, without any loss of generality and complexity, by directly differentiating the elliptic and hyperbolic grid generation equations. This results in a large system of equations which can be solved readily to yield the grid sensitivities. The technique developed is not restricted to two-dimensional problems and can be applied to three-dimensional problems without any additional effort. The developed grid sensitivity technique is then used in conjunction with the semianalytical aerodynamic sensitivity analysis procedure developed by Chattopadhyay and Pagaldipti [10], for calculating aerodynamic design sensitivities. In the following sections, the numerical scheme used in the flow analysis and the discrete semianalytical aerodynamic sensitivity approach described in [10] are outlined for the sake of completeness. This is followed by a detailed description of the semianalytical grid sensitivity analysis procedure.

FLOW SOLUTION

The parabolized Navier-Stokes (PNS) equations have been used for the evaluation of three-dimensional, supersonic, viscous flow fields. The assumptions made in deriving these equations are as follows. The streamwise derivatives of the viscous terms are neglected. The inviscid region of the flow field must be supersonic and the streamwise velocity component must be positive everywhere. Thus streamwise flow separation is not allowed, but crossflow separation is allowed. Efficiency in computational time and memory requirements are achieved because the equations can be solved using a space-marching technique. The PNS equations are usually considered in a generalized body fitted coordinate system. Let xyz be a rectangular Cartesian system fitted to the body with x being the longitudinal coordinate and y and z the normal coordinates. Let $\xi\eta\zeta$ be the body fitted grid coordinate system in which computation is performed, ξ being the streamwise coordinate, ζ, the coordinate normal to the body surface, and η, the circumferential coordinate (Figure 1). The PNS equations are written as [11]

$$\frac{\partial \overline{\mathbf{E}}_p}{\partial \xi} + \frac{\partial \overline{\mathbf{E}}_{pp}}{\partial \xi} + \frac{\partial \overline{\mathbf{F}}}{\partial \eta} + \frac{\partial \overline{\mathbf{G}}}{\partial \zeta} = \frac{\partial \overline{\mathbf{F}}_{\vartheta p}}{\partial \eta} + \frac{\partial \overline{\mathbf{G}}_{\vartheta p}}{\partial \zeta}, \tag{1}$$

where

$$\overline{\mathbf{E}}_p = \frac{1}{J}\left[\xi_x \mathbf{E}_p + \xi_y \mathbf{F}_p + \xi_z \mathbf{G}_p\right] \tag{2}$$

$$\overline{\mathbf{E}}_{pp} = \frac{1}{J}\left[\xi_x \mathbf{E}_{pp} + \xi_y \mathbf{F}_{pp} + \xi_z \mathbf{G}_{pp}\right] \tag{3}$$

$$\overline{\mathbf{F}} = \frac{1}{J}\left[\eta_x \mathbf{E} + \eta_y \mathbf{F} + \eta_z \mathbf{G}\right] \tag{4}$$

$$\overline{\mathbf{G}} = \frac{1}{J}\left[\zeta_x \mathbf{E} + \zeta_y \mathbf{F} + \zeta_z \mathbf{G}\right] \tag{5}$$

$$\mathbf{E}_p = \begin{bmatrix} \rho u & \rho u^2 + \omega p & \rho uv & \rho uw & (\rho e_t + p)\, u \end{bmatrix}^{\mathsf{T}} \tag{6}$$

$$\mathbf{F}_p = \begin{bmatrix} \rho v & \rho uv & \rho v^2 + \omega p & \rho vw & (\rho e_t + p)\, v \end{bmatrix}^{\mathsf{T}} \tag{7}$$

$$\mathbf{G}_p = \begin{bmatrix} \rho w & \rho uw & \rho vw & \rho w^2 + \omega p & (\rho e_t + p)\, w \end{bmatrix}^{\mathsf{T}} \tag{8}$$

$$\mathbf{E}_{pp} = \begin{bmatrix} 0 & (1-\omega)p & 0 & 0 & 0 \end{bmatrix}^{\mathsf{T}} \tag{9}$$

$$\mathbf{F}_{pp} = \begin{bmatrix} 0 & 0 & (1-\omega)p & 0 & 0 \end{bmatrix}^{\mathsf{T}} \tag{10}$$

$$\mathbf{G}_{pp} = \begin{bmatrix} 0 & 0 & 0 & (1-\omega)p & 0 \end{bmatrix}^{\mathsf{T}}. \tag{11}$$

In equations (6)–(11), ρ represents the density of the fluid, u, v, and w are the velocity components in the x, y, and z directions, respectively, p is the pressure, and ω is a parameter determined by stability analysis. The quantities $\mathbf{E}$, $\mathbf{F}$, and $\mathbf{G}$ are the inviscid flux vectors and are obtained by setting $\omega = 1$ in the expressions for $\mathbf{E}_p$, $\mathbf{F}_p$, and $\mathbf{G}_p$, respectively. The viscous flux vectors, $\mathbf{F}_{\nu p}$ and $\mathbf{G}_{\nu p}$, are presented in [11] and are not reproduced here. The partial derivatives, $\xi_x, \xi_y, \dots, \zeta_y, \zeta_z$, are the metrics of transformation between the $\xi\eta\zeta$ and the xyz system, and J denotes the Jacobian of transformation.

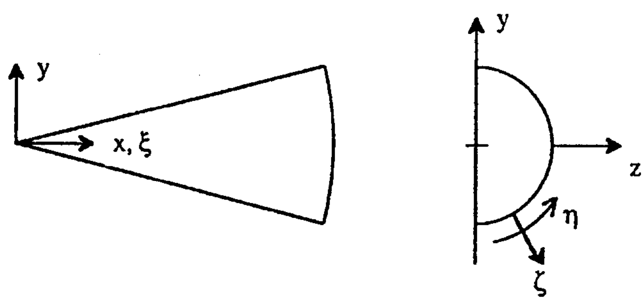

Figure 1. Coordinate systems.

In this paper, the PNS equations have been used for the efficient prediction of three-dimensional, steady, supersonic, viscous flow fields. The computational procedure used in this study, as implemented in the code UPS3D [14], integrates the PNS equations using an implicit, approximately factored, finite-volume algorithm where the crossflow inviscid fluxes are evaluated by Roe's flux-difference splitting scheme [15]. The UPS3D code also has the capability to calculate the inviscid flow field by solving the PNS equations without the viscous terms. The upwind algorithm is used to improve the resolution of the shock waves over that obtained with the conventional central differencing schemes. The UPS3D solver calculates the nondimensional force coefficients, such as lift coefficient (C_L) and drag coefficient (C_D) by integrating the pressure distributions over the surface of the body.

DISCRETE SEMIANALYTICAL SENSITIVITY ANALYSIS

In the present work, the UPS3D solver has been modified significantly to incorporate the calculation of the aerodynamic design sensitivities within its numerical procedure. The design sensitivity technique is outlined here. In general, an aerodynamic performance coefficient C_j depends on the steady-state flow variables Q^*, the vector of computational grid coordinates X, and sometimes, explicitly on the vector of independent design variables Φ. Mathematically,

$$C_j = C_j\left(Q^*(\Phi), X(\Phi), \Phi\right). \tag{12}$$

The derivative of C_j with respect to the i^{th} design variable ϕ_i is expressed as follows:

$$\frac{dC_j}{d\phi_i} = \left\{\frac{\partial C_j}{\partial Q^*}\right\}^{\mathsf{T}} \left\{\frac{\partial Q^*}{\partial \phi_i}\right\} + \left\{\frac{\partial C_j}{\partial X}\right\}^{\mathsf{T}} \left\{\frac{\partial X}{\partial \phi_i}\right\} + \frac{\partial C_j}{\partial \phi_i}. \tag{13}$$

In equation (13), the terms $\{\frac{\partial C_j}{\partial Q^*}\}$, $\{\frac{\partial C_j}{\partial X}\}$, and $\frac{\partial C_j}{\partial \phi_i}$ are easily calculated knowing the explicit dependence of C_j on Q^*, X, and ϕ_i. The term $\{\frac{\partial Q^*}{\partial \phi_i}\}$, which represents the sensitivity of the steady state flow variables with respect to the i^{th} design variable, is calculated using the direct differentiation technique. In the discrete sensitivity approach, the discretized flow equations are directly differentiated, as described next. The discretized flow equations which model the flow can be written as follows:

$$\{R(Q^*(\Phi), X(\Phi), \Phi)\} = \{0\}. \tag{14}$$

Equation (14), differentiated with respect to ϕ_i, yields

$$\left\{\frac{dR}{d\phi_i}\right\} = \left\{\frac{\partial R}{\partial Q^*}\right\}^{\mathsf{T}} \left\{\frac{\partial Q^*}{\partial \phi_i}\right\} + \left\{\frac{\partial R}{\partial X}\right\}^{\mathsf{T}} \left\{\frac{\partial X}{\partial \phi_i}\right\} + \frac{\partial R}{\partial \phi_i} = \{0\}. \tag{15}$$

Equation (15) represents a set of linear algebraic equations in $\frac{\partial Q^*}{\partial \phi_i}$ which can be solved easily. It is to be noted that the terms $\{\frac{\partial R}{\partial Q^*}\}$, $\{\frac{\partial R}{\partial X}\}$, and $\frac{\partial R}{\partial \phi_i}$ in equation (15) can be calculated easily knowing the explicit dependence of $\{R\}$ on Q^*, X, and ϕ_i.

GRID SENSITIVITY TECHNIQUE

The term $\{\frac{\partial X}{\partial \phi_i}\}$ appearing in equations (13) and (15) represents the grid sensitivity vector which must be computed semianalytically. The semianalytical grid sensitivity approach is illustrated on a hyperbolic grid generator first and is described for elliptic grid generators next. In general, a three-dimensional hyperbolic grid generation code generates a two-dimensional grid at various stations along the longitudinal direction by solving the following equations [11]:

$$\frac{\partial y \partial y}{\partial \eta \partial \zeta} + \frac{\partial z \partial z}{\partial \eta \partial \zeta} = 0 \tag{16}$$

$$\frac{\partial y \partial z}{\partial \eta \partial \zeta} - \frac{\partial z \partial y}{\partial \eta \partial \zeta} = F(\eta, \zeta). \tag{17}$$

In equation (17), $F(\eta,\zeta)$ is a known function approximating the Jacobian of transformation between the xyz and the $\xi\eta\zeta$ coordinate systems. The quantity $F(\eta,\zeta)$ is determined using a technique described in [16]. In this approach, the length of the inner boundary is drawn as a straight line with the same grid point distribution. Subsequently, parallel grid lines are created to produce a nonuniform grid spacing in a rectangular domain. The Jacobian is calculated to provide the cell area function $F(\eta,\zeta)$. Equations (16) and (17) are discretized and solved numerically to obtain the grid vector X. The grid sensitivity vector $\{\frac{\partial X}{\partial \phi_i}\}$ can be obtained by directly differentiating equations (16) and (17) with respect to ϕ_i after their discretization, as follows:

$$\frac{\partial y}{\partial \eta} \frac{\left(\frac{dy_{j+1,k}}{d\phi_i} - \frac{dy_{j,k}}{d\phi_i}\right)}{(\zeta_{j+1} - \zeta_j)} + \frac{\partial y \left(\frac{dy_{j,k+1}}{d\phi_i} - \frac{dy_{j,k}}{d\phi_i}\right)}{\partial \zeta\, (\eta_{k+1} - \eta_k)} + \frac{\partial z \left(\frac{dz_{j+1,k}}{d\phi_i} - \frac{dz_{j,k}}{d\phi_i}\right)}{\partial \eta\, (\zeta_{j+1} - \zeta_j)} + \frac{\partial z \left(\frac{dz_{j,k+1}}{d\phi_i} - \frac{dz_{j,k}}{d\phi_i}\right)}{\partial \zeta\, (\eta_{k+1} - \eta_k)} = 0 \tag{18}$$

$$\frac{\partial y}{\partial \eta} \frac{\left(\frac{dz_{j+1,k}}{d\phi_i} - \frac{dz_{j,k}}{d\phi_i}\right)}{(\zeta_{j+1} - \zeta_j)} + \frac{\partial z \left(\frac{dy_{j,k+1}}{d\phi_i} - \frac{dy_{j,k}}{d\phi_i}\right)}{\partial \zeta\, (\eta_{k+1} - \eta_k)} - \frac{\partial z \left(\frac{dy_{j+1,k}}{d\phi_i} - \frac{dy_{j,k}}{d\phi_i}\right)}{\partial \eta\, (\zeta_{j+1} - \zeta_j)} - \frac{\partial y \left(\frac{dz_{j,k+1}}{d\phi_i} - \frac{dz_{j,k}}{d\phi_i}\right)}{\partial \zeta\, (\eta_{k+1} - \eta_k)} = \frac{dF}{d\phi_i}. \tag{19}$$

Equations (18) and (19) represent a system of equations which can be solved readily to yield the grid sensitivity vector $\{\frac{\partial X}{\partial \phi_i}\}$.

In a similar fashion, a three-dimensional elliptic grid generation code generates a two-dimensional grid at various stations along the longitudinal direction by solving the following equations [11]:

$$ax_{\xi\xi} - 2bx_{\xi\eta} + cx_{\eta\eta} = \frac{-1}{J^2}\left(Px_\xi + Qx_\eta\right) \tag{20}$$

$$ay_{\xi\xi} - 2by_{\xi\eta} + cy_{\eta\eta} = \frac{-1}{J^2}\left(Py_\xi + Qy_\eta\right), \tag{21}$$

where

$$a = x_\eta^2 + y_\eta^2 \tag{22}$$

$$b = x_\xi x\eta + y_\xi y_\eta \tag{23}$$

$$c = x_\xi^2 + y_\xi^2. \tag{24}$$

In equations (20) and (21), $P(\eta,\zeta)$ and $Q(\eta,\zeta)$ are known function controlling grid clustering and orthogonality, and J is the Jacobian of transformation. Equations (20) and (21) are discretized and solved numerically to obtain the grid vector X. Since equations (20) and (21) are nonlinear, they are linearized by evaluating the coefficients from the previous iteration [11]. The grid sensitivity vector $\{\frac{\partial X}{\partial \phi_i}\}$ can then be obtained by directly differentiating equations (20) and (21) with respect to ϕ_i after their discretization and by solving the resulting system of algebraic equations in the grid sensitivities.

The grid sensitivity and the aerodynamic sensitivity techniques yield large systems of algebraic equations characterized by sparse coefficient matrices. Direct methods for solving these systems of equations are inefficient and iterative techniques have been used in this work. The systems of equations are solved using the successive over relaxation scheme (SOR). A detailed description of this iterative technique can be found in [11] and is not repeated here.

RESULTS

Results obtained from the grid sensitivity technique developed are presented here. The technique is first applied to a two-dimensional elliptic grid around a circular arc airfoil (Figure 2). The circular arc airfoil has a unit chord length and thickness-to-chord ratio of 0.06. The computational grid includes 39 grid points in the streamwise direction and 25 grid points in the normal direction. Grid clustering and orthogonality are neglected. The derivatives of the (x, y) coordinates of the grid points with respect to the airfoil chord length (c) and thickness-to-chord ratio (t_c), calculated using the developed technique and the finite difference method, are compared at two representative grid points in Table 1. It is clearly seen that the sensitivities obtained from both the techniques agree very well. Figure 3 compares the CPU time required by the direct differentiation technique and the finite difference approach. In this case, an 8 percent reduction in CPU time is observed by using the direct differentiation grid sensitivity technique. The grid sensitivity technique is next applied to a three-dimensional hyperbolic grid around a wing-body configuration (Figure 4). The wing root chord (c_o), leading edge sweep (λ), wing span (w_s), and the wing thickness-to-chord ratio (t_c) are chosen as design variables. The computational grid includes 75 grid points in the circumferential direction, 1600 grid points in the longitudinal direction and 40 grid points in the normal direction (Figure 5). The grid sensitivities of two representative grid points, calculated using the direct differentiation technique and finite difference technique, are presented in Table 2. As shown, there is excellent agreement between the two techniques. Further, a comparison of the CPU time shows a 42 percent reduction achieved for one complete grid sensitivity analysis using the developed procedure (Figure 6). The reduction in the CPU time

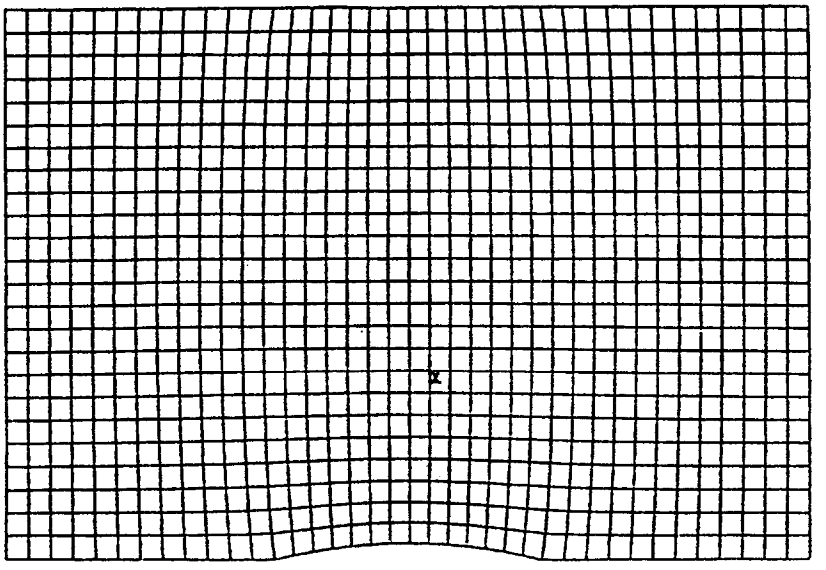

Figure 2. Elliptic grid around a circular arc airfoil.

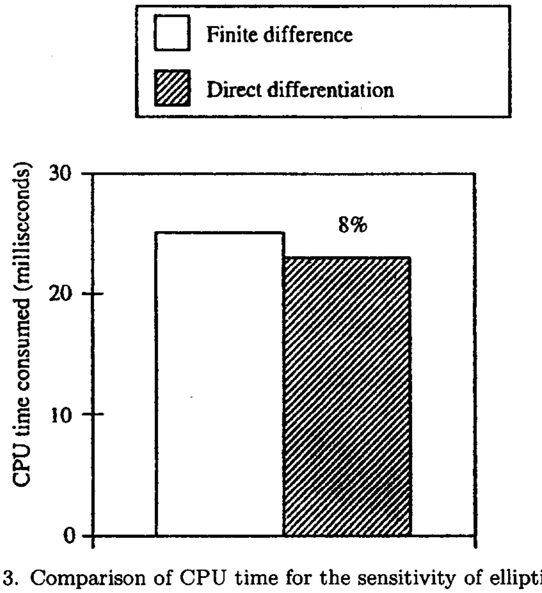

Figure 3. Comparison of CPU time for the sensitivity of elliptic grid.

Table 1. Sensitivity of the two-dimensional elliptic grid, $\frac{dx}{d\phi_i}$ and $\frac{dy}{d\phi_i}$.

Grid point (x, y)	Design variable (ϕ_i)	Finite difference grid sensitivity method	Direct differentiation grid sensitivity method
$(0.0, 0.06)$	Chord (c)	$(0.0, 0.12)$	$(0.0, 0.12)$
	Thickness/chord (t_c)	$(0.0, 1.00)$	$(0.0, 1.00)$
$(-0.5, 0.00)$	Chord (c)	$(-0.5, 0.00)$	$(-0.5, 0.00)$
	Thickness/chord (t_c)	$(0.0, 0.00)$	$(0.0, 0.00)$

for the hyperbolic grid sensitivity is higher than that for the elliptic grid sensitivity due to several reasons. The elliptic grid sensitivity analysis has been performed on a two-dimensional elliptic grid whereas the hyperbolic grid sensitivity analysis has been performed for a three-dimensional hyperbolic grid. Further, the number of grid points used in the hyperbolic grid sensitivity analysis is much higher than that used in the elliptic grid sensitivity analysis. Most importantly, the elliptic grid sensitivity analysis uses two design variables, and the hyperbolic grid sensitivity analysis

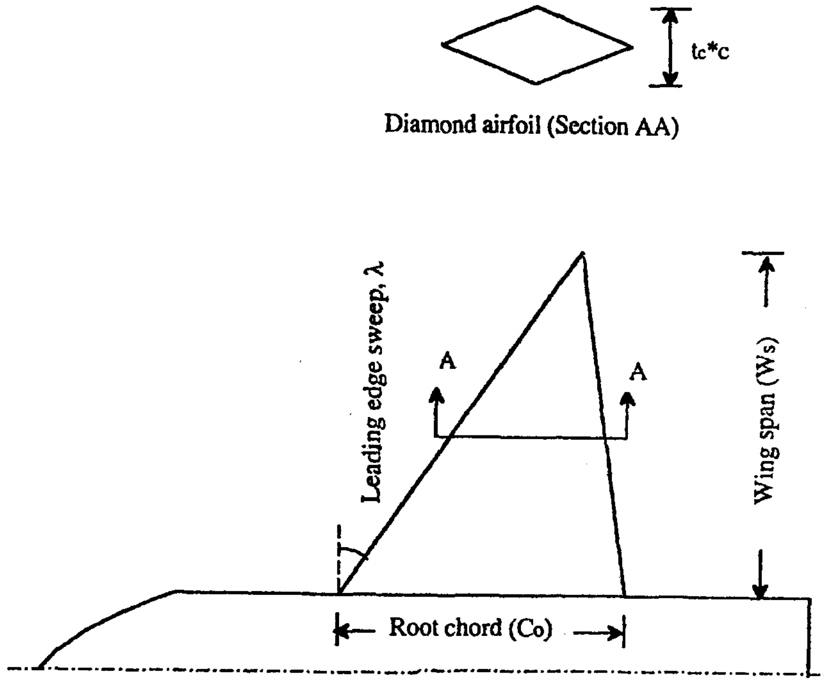

Figure 4. Design variables for the delta wing-body configuration.

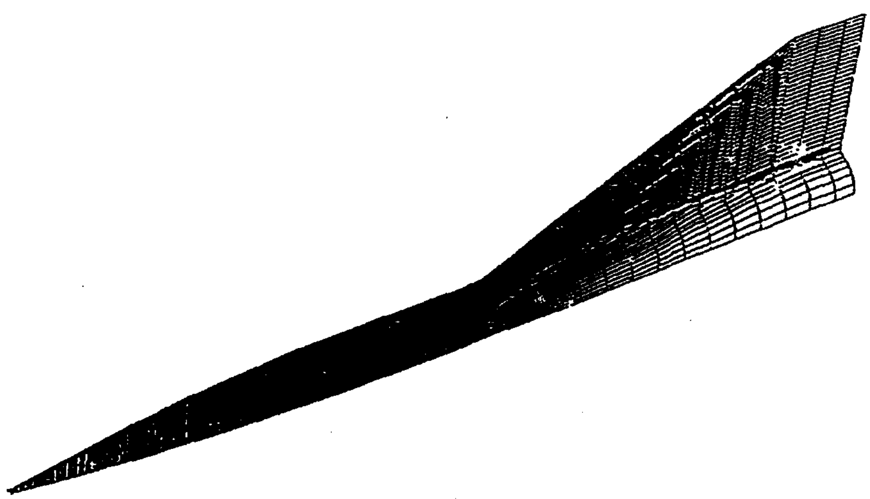

Figure 5. Surface grid for the wing-body configuration.

Table 2. Sensitivity of the three-dimensional hyperbolic grid $\frac{dx}{d\phi_i}$, $\frac{dy}{d\phi_i}$, and $\frac{dz}{d\phi_i}$.

Grid point (x, y, z)	Design variable (ϕ_i)	Finite difference grid sensitivity method	Direct differentiation grid sensitivity method
$(0.300, 0.044, 0.008)$	Sweep (λ)	$(0.0, 0.0, 0.0)$	$(0.0, 0.0, 0.0)$
	Root chord (c_o)	$(0.0, 0.0, 0.0)$	$(0.0, 0.0, 0.0)$
	Wing span (w_s)	$(0.0, 0.0, 0.0)$	$(0.0, 0.0, 0.0)$
	Thickness/chord (t_c)	$(0.0, 0.0, 0.0)$	$(0.0, 0.0, 0.0)$
(16.406, 13.297, 2.753)	Sweep (λ)	$(0.0, 0.0338, 0.3930)$	$(0.0, 0.0338, 0.3930)$
	Root chord (c_o)	$(0.0, 0.2886, 0.3356)$	$(0.0, 0.2884, 0.3355)$
	Wing span (w_s)	$(0.0, 0.8014, 0.9320)$	$(0.0, 0.8013, 0.9319)$
	Thickness/chord (t_c)	$(0.0, 46.640, 54.240)$	$(0.0, 46.550, 54.132)$

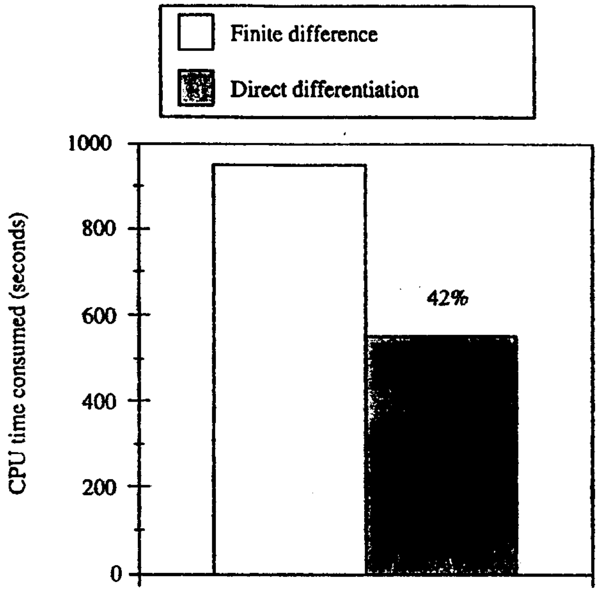

Figure 6. Comparison of CPU time for the sensitivity of hyperbolic grid.

uses four design variables. These factors result in a higher percentage of computational savings in the hyperbolic grid sensitivity example. Therefore, a direct comparison should not be made between the CPU savings in these two examples. The grid sensitivity analysis procedures clearly demonstrate the significant computational savings achievable by using this approach instead of the finite difference method, especially in a formal design optimization procedure where several such DSA are necessary.

Results obtained from the aerodynamic sensitivity analysis procedure are presented next. The wing-body configuration (Figure 4) of an advanced high speed aircraft, operating at a flight Mach number of 2.5 and an angle of attack of 5 degrees, is used as a reference design. The sensitivities of the drag coefficient (C_D) and the lift coefficient (C_L), calculated using the direct differentiation technique as well as the finite difference technique, are presented in Tables 3 and 4, respectively. It must be noted that column 3 in Tables 3 and 4 presents the results of the semianalytical aerodynamic sensitivity approach with finite difference grid sensitivity while column 4 presents the results of the semianalytical aerodynamic sensitivity approach with semianalytical grid sensitivity. As shown, the results from both techniques are in excellent agreement. For one complete sensitivity analysis, the direct differentiation technique with finite difference grid sensitivity calculations results in a 30 percent reduction in computing time from the finite difference technique (Figure 7). The semianalytical sensitivity analysis technique with semianalytical grid sensitivity calculations yields a 45 percent reduction in computing time from the finite difference approach (Figure 7). This further illustrates the efficiency of the discrete semianalytical technique for grid sensitivity calculations. In order to investigate the trend in CPU savings, the results are plotted for varying number of design variables. Figure 8 compares the computing time required for one complete sensitivity analysis, using the finite difference aerodynamic DSA, the semianalytical aerodynamic DSA using finite difference grid sensitivity, and the semianalytical aerodynamic DSA using semianalytical grid sensitivity as a function of the number of design variables. It is observed that, although all three curves follow a near linear relationship with the number of design variables, their slopes are quite different. The DSA with semianalytical grid sensitivity calculations results in the minimum slope. This indicates that, although, as expected, the computation time increases almost linearly with the increase in the number of design variables, the increase is less significant in the semianalytical approaches and least in the case where analytical grid sensitivities are used. This further illustrates the efficiency of the grid sensitivity analysis technique.

Table 3. Sensitivity of the drag coefficient ($\frac{dC_D}{d\phi_i}$).

Design variable	Finite difference aerodynamic sensitivity analysis	Aerodynamic sensitivity analysis using finite difference grid sensitivity	Aerodynamic sensitivity analysis using semi-analytical grid sensitivity
Sweep (λ)	−0.0097712	−0.0106882	−0.0106453
Root chord (c_o)	0.0356022	0.0353581	0.0353786
Wing span (w_s)	0.0239097	0.0210748	0.0229120
Thickness/chord (t_c)	1.4250140	1.4219202	1.4820189

Table 4. Sensitivity of the lift coefficient ($\frac{dC_L}{d\phi_i}$).

Design variable	Finite difference aerodynamic sensitivity analysis	Aerodynamic sensitivity analysis using finite difference grid sensitivity	Aerodynamic sensitivity analysis using semi-analytical grid sensitivity
Sweep (λ)	−0.0943048	−0.0920100	−0.0880802
Root chord (c_o)	0.0775282	0.0727289	0.0755285
Wing span (w_s)	0.0695290	0.0628465	0.0647937
Thickness/chord (t_c)	−1.8981815	−1.7628147	−1.8495492

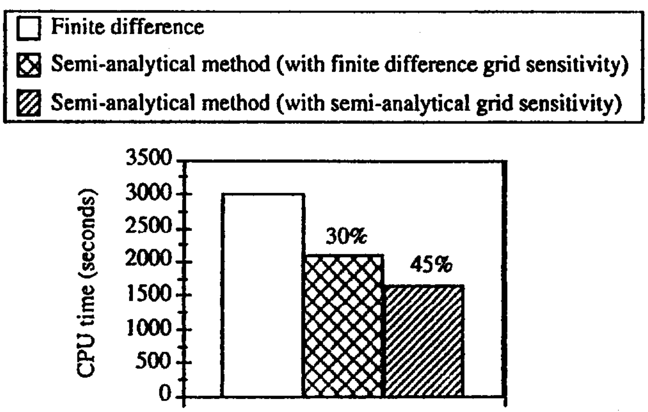

Figure 7. Comparison of CPU time for aerodynamic sensitivity analysis.

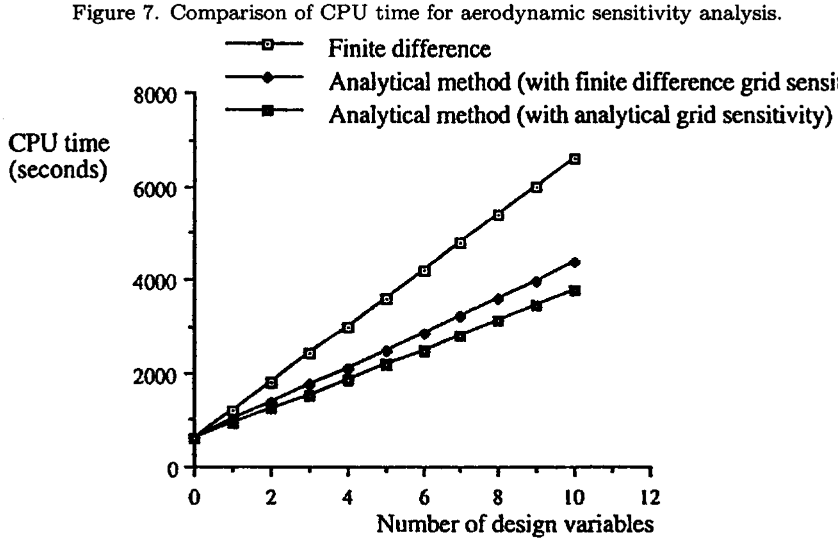

Figure 8. Comparison of CPU time with design variables.

CONCLUDING REMARKS

A discrete semianalytical sensitivity analysis procedure, based on the direct differentiation technique, has been developed for calculating aerodynamic sensitivities. A new approach has been developed for calculating grid sensitivities and has been demonstrated using an elliptical grid and a hyperbolic grid. The procedure is implemented within a semianalytical aerodynamic sensitivity analysis procedure. The following important observations are made:

1. The results from the semianalytical grid sensitivity technique compare very well with those obtained using a finite difference approach.
2. The developed grid sensitivity analysis procedures yield significant savings in computing time over the finite difference technique.
3. When used within a semianalytical aerodynamic sensitivity analysis technique, the procedure yielded accurate results with very significant CPU savings.
4. The semianalytical aerodynamic sensitivity technique in conjunction with the developed grid sensitivity procedure showed the minimum increase in CPU time with increase in total number of design variables.

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